Lesson 4: Construct a Perpendicular Bisector

Classwork

Opening Exercise

Choose **one** method below to check your homework:

- Trace your copied angles and bisectors onto patty paper, then fold the paper along the bisector you constructed. Did one ray exactly overlap the other?
- Work with your partner. Hold one partner's work over another's. Did your angles and bisectors coincide perfectly?

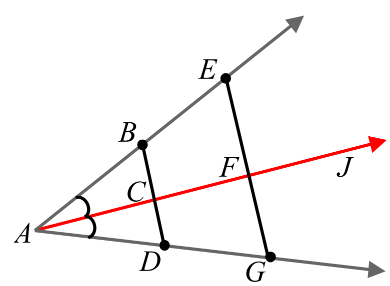
Use the following rubric to evaluate your homework:

Needs Improvement	Satisfactory	Excellent
Few construction arcs visible	Some construction arcs visible	Construction arcs visible and appropriate
Few vertices or relevant intersections labeled	Most vertices and relevant intersections labeled	All vertices and relevant intersections labeled
Lines drawn without straightedge or not drawn correctly	Most lines neatly drawn with straightedge	Lines neatly drawn with straightedge
Fewer than 3 angle bisectors constructed correctly	3 of the 4 angle bisectors constructed correctly	Angle bisector constructed correctly

Discussion

In Lesson 3 we studied how to construct an angle bisector. We know we can verify the construction by folding an angle along the bisector. A correct construction means one half of the original angle will coincide exactly with the other half so that each point of one ray of the angle maps onto a corresponding point on the other ray of the angle.

We now extend this observation. Imagine a segment that joins any pair of points that map onto each other when the original angle is folded along the bisector. The following figure illustrates two such segments:

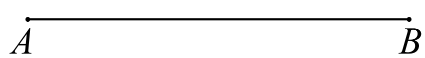


Let us examine one of the two segments, EG. When the angle is folded along ray $\overrightarrow{AJ}$, E coincides with G. In fact, folding the angle demonstrates that E is the same distance from F as G is from F; $EF = FG$. The point that separates these equal halves of EG is F, which is in fact the midpoint of the segment and lies on the bisector $\overrightarrow{AJ}$. We can make this case for any segment that falls under the conditions above.

By using geometry facts we acquired in earlier school years, we can also show that the angles formed by the segment and the angle bisector are right angles. Again, by folding, we can show that $\angle EFJ$ and $\angle GFJ$ coincide and must have the same measure. The two angles also lie on a straight line, which means they sum to 180˚. Since they are equal in measure and they sum to 180˚, they each have a measure of 90˚.

These arguments lead to a remark about symmetry with respect to a line, and the definition of a perpendicular bisector. Two points are symmetric with respect to a line l if and only if l is the perpendicular bisector of the segment that joins the two points. The **perpendicular bisector** of a segment AB is the line _____________________ to AB and passing through the _____________________ of AB.

We now investigate how to construct a perpendicular bisector of a line segment using a compass and straightedge. Using what you know about the construction of an angle bisector, experiment with your construction tools and the following line segment to establish the steps that determine this construction.



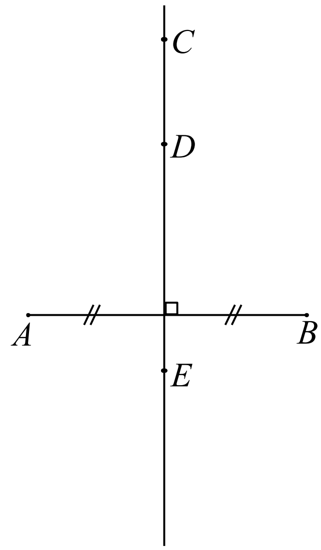
Precisely describe the steps you took to bisect the segment.

Now that you are familiar with the construction of a perpendicular bisector, we must make one last observation. Using your compass, string, or patty paper, examine the following pairs of segments:

I. AC, BC

II. AD, BD

III. AE, BE



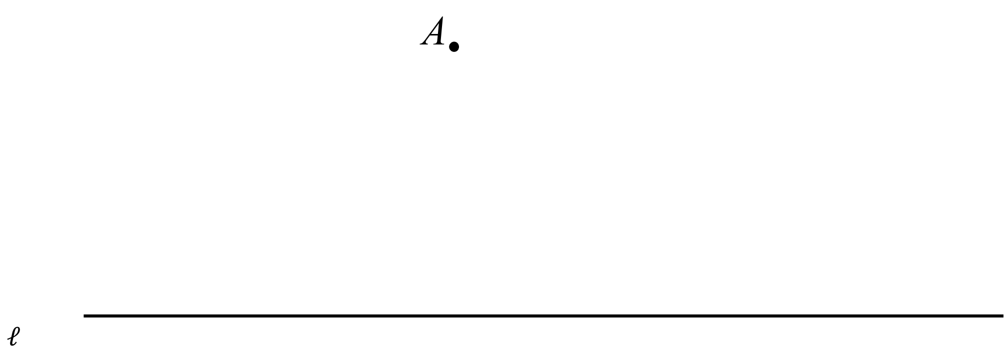
Based on your findings, fill in the observation below.

Observation:

Any point on the perpendicular bisector of a line segment is ______________________ from the endpoints of the line segment.

Mathematical Modeling Exercise

You know how to construct the perpendicular bisector of a segment. Now you will investigate how to construct a perpendicular to a line ℓ from a point A not on ℓ. Think about how you have used circles in constructions so far and *why* the perpendicular bisector construction works the way it does. The first step of the instructions has been provided for you. Discover the construction and write the remaining steps.



Step 1. Draw circle A so that the circle intersects line ℓ in two points.

Relevant Vocabulary

Right Angle: An angle is called a *right angle* if its measure is 90°.

Perpendicular: Two lines are *perpendicular* if they intersect in one point, and any of the angles formed by the intersection of the lines is a 90° angle. Two segments or rays are perpendicular if the lines containing them are perpendicular lines.

Equidistant: A point A is said to be *equidistant* from two different points B and C if $AB = AC$. A point A is said to be *equidistant* from a point B and a line L if the distance between A and L is equal to AB.

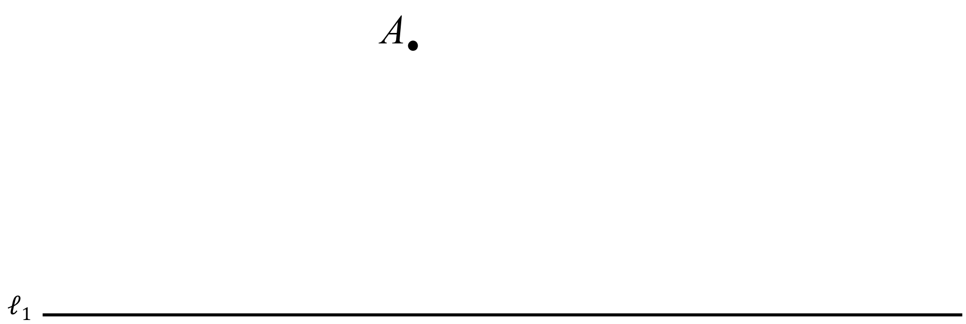
Problem Set

1. During this lesson, you constructed a perpendicular line to a line $\boldsymbol{\ell}$ from a point $\boldsymbol{A}$ not on $\boldsymbol{\ell}$. We are going to use that construction to construct parallel lines:

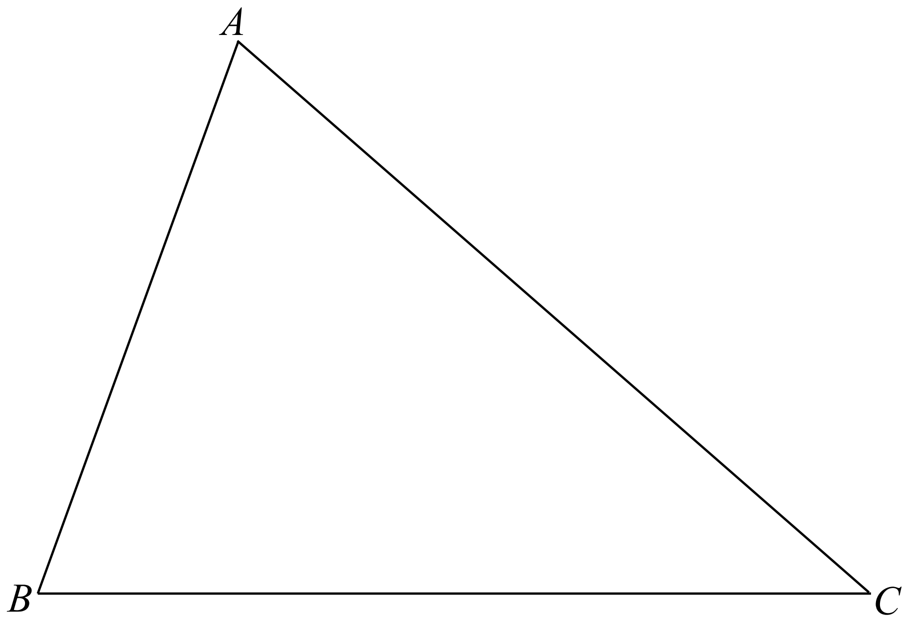
   To construct parallel lines ℓ_1 and ℓ_2:

   I. Construct a perpendicular line ℓ_3 to a line ℓ_1 from a point A not on ℓ_1.

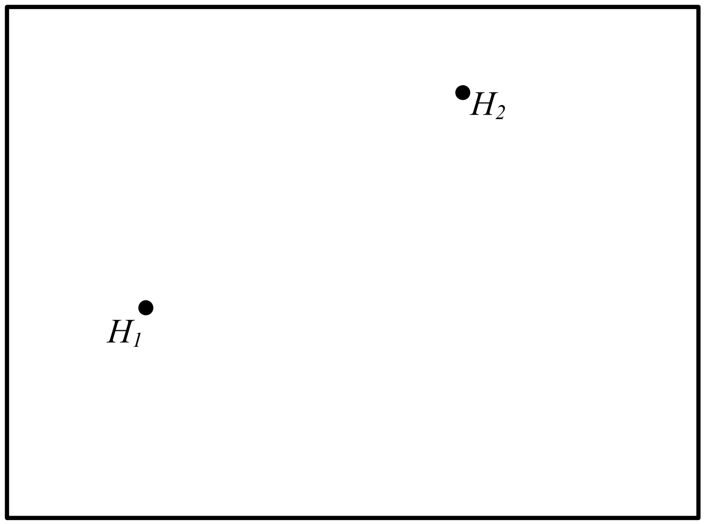
   II. Construct a perpendicular line ℓ_2 to ℓ_3 through point A. *Hint:* Consider using the steps behind Lesson 3, Problem Set #4 to accomplish this.



2. Construct the perpendicular bisector of AB, BC, and CA on the triangle below. What do you notice about the segments you have constructed?



3. Two homes are built on a plot of land. Both homeowners have dogs, and are interested in putting up as much fencing as possible between their homes on the land, but in a way that keeps the fence equidistant from each home. Use your construction tools to determine where the fence should go on the plot of land.



How will the fencing alter with the addition of a third home?