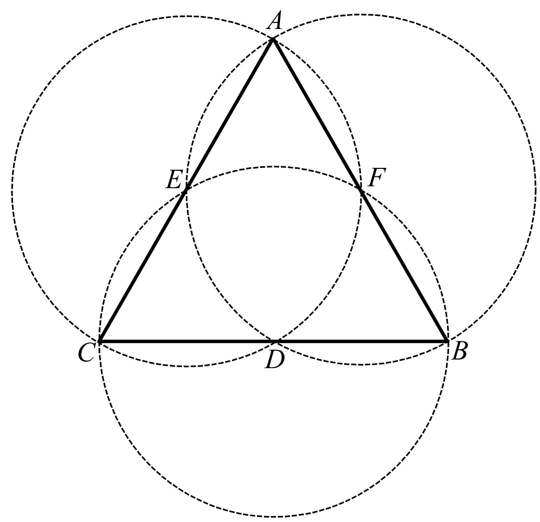
Lesson 3: Copy and Bisect an Angle

Classwork

Opening Exercise

In the following figure, circles have been constructed so that the endpoints of the diameter of each circle coincide with the endpoints of each segment of the equilateral triangle.



a. What is special about points D, E, and F? Explain how this can be confirmed with the use of a compass.

b. Draw DE, EF, and FD. What kind of triangle must $\triangle \mathrm{DEF}$ be?

c. What is special about the four triangles within $\triangle \mathrm{ABC}$?

d. How many times greater is the area of $\triangle \mathrm{ABC}$ than the area of $\triangle \mathrm{CDE}$?

Discussion

Define the terms **angle, interior of an angle,** and **angle bisector**.

Angle: An *angle* is ____

____

____

Interior: The *interior of angle* $\angle BAC$ is the set of points in the intersection of the half-plane of AC that contains B and the half-plane of AB that contains C. The interior is easy to identify because it is always the "smaller" region of the two regions defined by the angle (the region that is convex). The other region is called the *exterior* of the angle.

Angle Bisector: If C is in the interior of $\angle AOB$, ____

____

____

When we say $\angle \boldsymbol{AOC} = \angle \boldsymbol{COB}$, we mean that the angle measures are equal and that $\angle \boldsymbol{AOC}$ can either refer to the angle itself or its measure when the context is clear.

Geometry Assumptions

In working with lines and angles, we again make specific assumptions that need to be identified. For example, in the definition of interior of an angle above, we assumed that angle separated the plane into two disjoint sets. This follows from the assumption: *Given a line, the points of the plane that do not lie on the line form two sets called half-planes, such that (1) each of the sets is convex and (2) if P is a point in one of the sets, and Q is a point in the other, then the segment $\overline{PQ}$ intersects the line.*

From this assumption another obvious fact follows about a segment that intersects the sides of an angle: *Given an angle $\angle AOB$, then for any point C in the interior of $\angle AOB$, the ray $\overrightarrow{OC}$ will definitely intersect the segment $\overline{AB}$.*

In this lesson, we move from working with line segments to working with angles– specifically with bisecting angles. Before we do this, we need to clarify our assumptions about measuring angles. These assumptions are based upon what we know about a protractor that measures up to 180˚ angles:

1. To every angle $\angle AOB$ there corresponds a real number $|\angle AOB|$ called the degree or measure of the angle so that $0 < |\angle AOB| < 180$.

This number, of course, can be thought of as the angle measurement (in degrees) of the interior part of the angle, which is what we read off of a protractor when measuring an angle. In particular, we have also seen that we can use protractors to "add angles":

2. If C is a point in the interior of $\angle AOB$, then $|\angle AOC| + |\angle COB| = |\angle AOB|$. (Abbreviation: $\angle s$ add.)

Two angles $\angle BAC$ and $\angle CAD$ form a *linear pair* if $\overrightarrow{AB}$ and $\overrightarrow{AD}$ are opposite rays on a line, and $\overrightarrow{AC}$ is any other ray. In earlier grades, we abbreviated this situation and the fact that the angles on a line add up to 180˚ as, "$\angle s$ on a line." Now we state it formally as one of our assumptions:

3. If two angles $\angle BAC$ and $\angle CAD$ form a linear pair, then they are supplementary, i.e., $|\angle BAC| + |\angle CAD| = 180$. (Abbreviation: $\angle s$ on a line.)

Protractors also help us to draw angles of a specified measure:

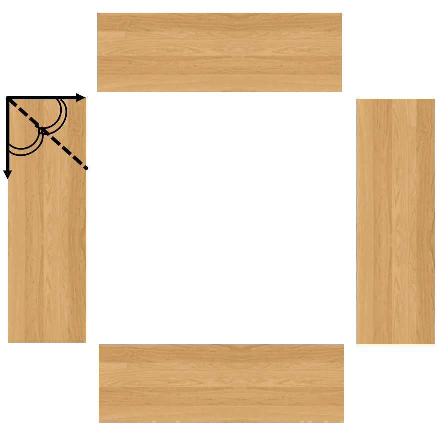
4. Let $\overrightarrow{OB}$ be a ray on the edge of the half-plane H. For every r such that $0 < r < 180$, there is exactly one ray $\overrightarrow{OA}$ with A in H such that $|\angle AOB| = r$.

Example 1: Investigate How to Bisect an Angle

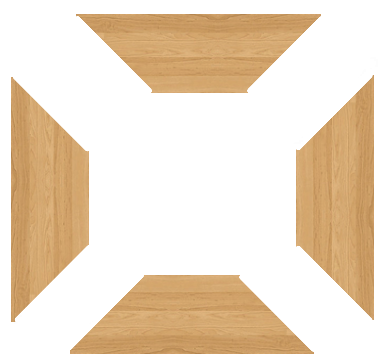
You will need a compass and a straightedge.

Joey and his brother, Jimmy, are working on making a picture frame as a birthday gift for their mother. Although they have the wooden pieces for the frame, they need to find the angle bisector to accurately fit the edges of the pieces together. Using your compass and straightedge, show how the boys bisected the corner angles of the wooden pieces below to create the finished frame on the right.

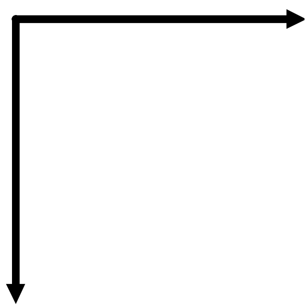
Before

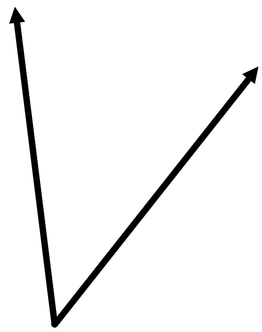


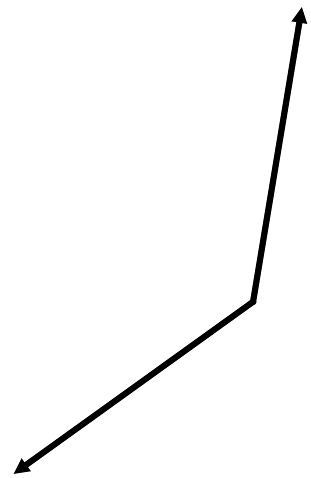
After



Consider how the use of circles aids the construction of an angle bisector. Be sure to label the construction as it progresses and to include the labels in your steps. Experiment with the angles below to determine the correct steps for the construction.





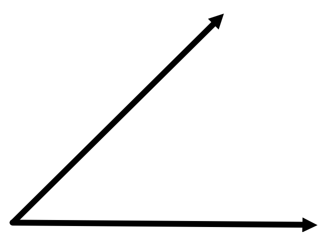


What steps did you take to bisect an angle? List the steps below:

Example 2: Investigate How to Copy an Angle

You will need a compass and a straightedge.

You and your partner will be provided with a list of steps (in random order) needed to copy an angle using a compass and straightedge. Your task is to place the steps in the correct order, then follow the steps to copy the angle below.



Steps needed (in correct order):

1. ______
2. ______
3. ______
4. ______
5. ______
6. ______
7. ______
8. ______
9. ______

Relevant Vocabulary

Midpoint: A point B is called a *midpoint* of a segment $\overline{AC}$ if B is between A and C, and $AB = AC$.

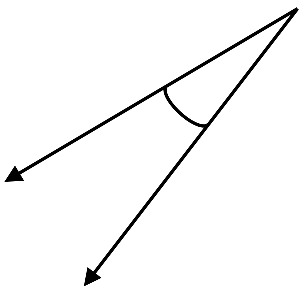
Degree: Subdivide the length around a circle into 360 arcs of equal length. A central angle for any of these arcs is called a *one-degree angle* and is said to have angle measure 1 degree. An angle that turns through n one-degree angles is said to have an angle measure of n degrees.

Zero and Straight Angle: A *zero angle* is just a ray and measures 0˚. A *straight angle* is a line and measures 180˚ (the ˚ is an abbreviation for "degree").

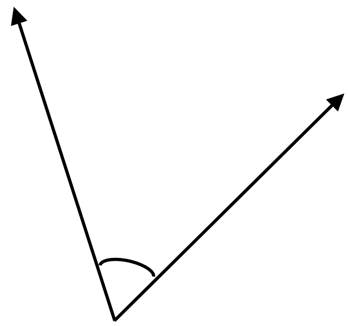
Problem Set

Directions: Bisect each angle below.

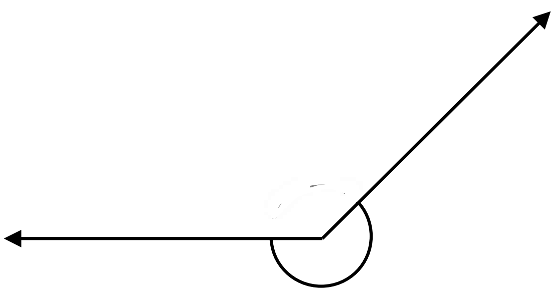
1.



2.



3.



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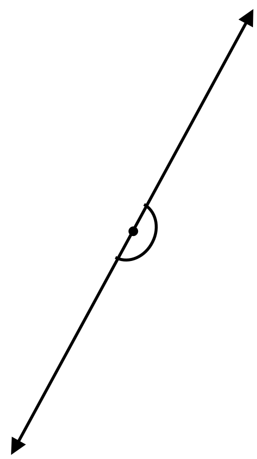

Directions: Copy the angle below.

5.