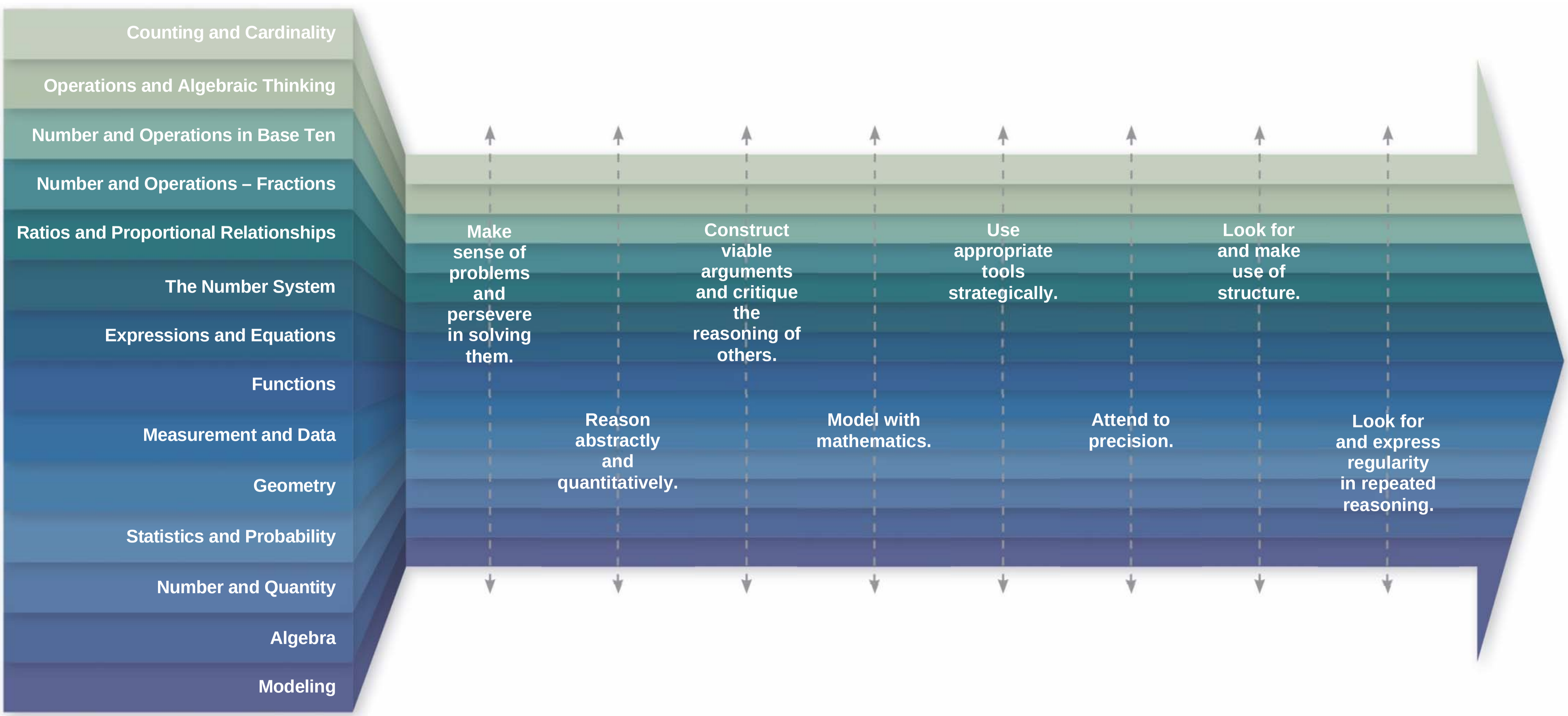
NEW YORK STATE EDUCATION DEPARTMENT

New York State Next Generation
Mathematics Learning Standards

2017



Introduction

In 2015, New York State (NYS) began a process of review and revision of its current mathematics standards adopted in January of 2011. Through numerous phases of public comment, virtual and face-to-face meetings with committees consisting of NYS educators (Special Education, Bilingual Education and English as a New Language teachers), parents, curriculum specialists, school administrators, college professors, and experts in cognitive research, the *New York State Next Generation Mathematics Learning Standards (2017)* were developed. These revised standards reflect the collaborative efforts and expertise of all constituents involved.

The *New York State Next Generation Mathematics Learning Standards (2017)* reflect revisions, additions, vertical movement, and clarifications to the current mathematics standards. The Standards are defined as the knowledge, skills and understanding that individuals can and do habitually demonstrate over time because of instruction and learning experiences. These mathematics standards, collectively, are focused and cohesive—designed to support student access to the knowledge and understanding of the mathematical concepts that are necessary to function in a world very dependent upon the application of mathematics, while providing educators the opportunity to devise innovative programs to support this endeavor. As with any set of standards, they need to be rigorous; they need to demand a balance of conceptual understanding, procedural fluency and application and represent a significant level of achievement in mathematics that will enable students to successfully transition to post-secondary education and the workforce.

Context for Revision of the *NYS Next Generation Mathematics Learning Standards (2017)*

Changing expectations for mathematics achievement

Today's children are growing up in a world very different from the one even 15 years ago. Seismic changes in the labor market mean that we are living and working in a knowledge-based economy—one that demands advanced literacy and Science, Technology, Engineering and Mathematics (STEM) skills, whether for application in the private or public sector. Today, information moves through media at lightning speeds and is accessible in ways that are unprecedented; technology has eliminated many jobs while changing and creating others, especially those involving mathematical and conceptual reasoning skills. One characteristic of these fast-growing segment of jobs is that the employee needs to be able to solve unstructured problems while working with others in teams. At the same time, migration and immigration rates around the world bring diversity to schools and neighborhoods. The exponential growth in interactions and information sharing from around the world means there is much to process, communicate, analyze and respond to in the everyday, across all settings. For a great majority of jobs, conceptual reasoning and technical writing skills are integral parts to the daily routine.

To prepare students for the changes in the way we live and work, and to be sure that our education system keeps pace with what it means to be mathematically literate and what it means to collaboratively problem solve, we need a different approach to daily teaching and learning. We need content-rich standards that will serve as a platform for advancing children's 21st-century mathematical skills —their abstract reasoning, their collaboration skills, their ability to learn from peers and through technology, and their flexibility as a learner in a dynamic learning environment. Students need to be engaged in dialogue and learning experiences that allow complex topics and ideas to be explored from many angles and perspectives. They also need to learn how to think and solve problems for which there is no one solution—and learn mathematical skills along the way.

Increasingly Diverse Learner Populations

The need for a deeper, more innovative approach to mathematics teaching comes at a time when the system is already charged with building up language skills among the increasingly diverse population. Students who are English Language Learners (ELLs)/Multilingual Learners (MLLs) now comprise over 20% of the school-age population, which reflects significant growth in the past several decades. Between 1980 and 2009, this population increased from 4.7 to 11.2 million young people, or from 10 to 21% of the school-age population. This growth will likely continue in U.S. schools; by 2030, it is anticipated that 40% of the school-age population in the U.S. will speak a language other than English at home.[(1)] Today, in schools and districts across the U.S., many students other than those classified as ELLs are learning English as an additional language, even if not in the initial stages of language development—these children are often described as "language minority learners." Likewise, many students, large numbers of whom are growing up in poverty, speak a dialect of English that is different from the academic English found in school curriculum.[(2),(3),(4)] Each of these groups—ELLs/MLLs, language minority learners, and students acquiring academic English—often struggle to access the language, and therefore the knowledge that fills the pages of academic texts, despite their linguistic assets. Therefore, the context for this new set of Mathematics Standards is that there is a pressing need to provide instruction that not only meets, but exceeds standards, as part of system-wide initiative to promote equal access to math skills for all learners while capitalizing on linguistic and cultural diversity.

All academic work does, to some degree, involve the academic language needed for success in school. For many students, including ELLs/MLLs, underdeveloped academic language affects their ability to comprehend and analyze texts, limits their ability to write and express their mathematical reasoning effectively, and can hinder their acquisition of academic content in all academic areas in which learning is demonstrated and assessed through oral and written language. If there isn't sufficient attention paid to building academic language across all content areas, students, including ELLs/MLLs, will not reach their potential and we will continue to perpetuate achievement gaps. The challenge is to design instruction that acknowledges the role of language; because language and knowledge are so inextricable.

In summary, today's children live in a society where many of their peers are from diverse backgrounds and speak different languages; one where technology is ubiquitous and central to daily life. They will enter a workforce and economy that demands critical thinking skills, and strong communication and social skills for full participation in society. This new society and economy has implications for today's education system—especially our instruction to foster a deeper and different set of communication and critical thinking skills, with significant attention to STEM.

Students with Disabilities and the Standards

One of the fundamental tenets guiding educational legislation (the *No Child Left Behind Act*, and *Every Student Succeeds Act*), and related policies over the past 15-years, is that all students, including students with disabilities, can achieve high standards of academic performance. A related trend is the increasing knowledge and skill expectations for PreK-Grade 12 students, especially in the area of reading and language arts, required for success in postsecondary education and 21st Century careers. Indeed, underdeveloped literacy skills have profound academic, social, emotional, and economic consequences for students, families, and society.

At the same time, the most recently available federal data[(5)] presents a portrait of the field reflecting both challenges and opportunities.

- *Students served under IDEA, Part B*: During the 2012-13 school year, there was a total of 5.83 million students with disabilities, ages 6-21; an increase from 5.67 million in 2010-11.
- *Access to the general education program*: More than 60 percent (62.1%) of students, ages 6 through 21 served under IDEA, Part B, were educated in the regular classroom 80% or more of the day, up from 60.5% in 2010-11.
- *Participation in state assessments*: Between 68.1 and 84.1 percent of students with disabilities in each of grades 3 through 8 and high school participated in the regular state assessment in reading based on grade-level academic achievement standards with or without accommodations.
- *English language arts proficiency*: The median percentages of students with disabilities in grades 3 through 8 and high school who were administered the 2012-13 state assessment in reading based on grade-level academic achievement standards who were proficient ranged from 25.4 to 37.3 percent.
- *Graduation*: Over sixty percent (65.1%) of students with disabilities graduated with a regular high school diploma.

Overall, the number of students with disabilities is increasing nationwide, as is their access to the general education curriculum, and participation in the state ELA and mathematics assessments. Attaining proficiency and graduating with a regular high school diploma are areas where significant improvements are needed.

Therefore, each student's individualized education program (IEP) must be developed in consideration of the State learning standards and should include information for teachers to effectively provide supports and services to address the individual learning needs of the student as they impact the student's ability to participate and progress in the general education curriculum. In addition to supports and services, special education must include specially designed instruction, which means adapting, as appropriate, the content, methodology or delivery of instruction to address the unique needs that result from the student's disability. By so doing, the teacher ensures each student's access to the general education curriculum so that he or she can meet the learning standards that apply to all students. The *Blueprint for Improved Results for Students with Disabilities* focuses on seven core evidence-based principles for students with disabilities to ensure they have the opportunity to benefit from high quality instruction and to reach the same academic standards as all students. For additional information, please see the Office of Special Education's field advisory: *Blueprint for Improved Results for Students with Disabilities*.

Understanding the *NYS Next Generation Mathematics Learning Standards (2017)*

The *NYS Next Generation Mathematics Learning Standards (2017)* define what students should understand and be able to do as a result of their study of mathematics. To assess progress on the Standards, a teacher must assess whether the student has understood what has been taught and provide opportunities where a student can independently use and apply this knowledge to solve mathematical problems in similar or new contexts. While procedural skills are relatively straightforward to assess, teachers often ask: what does mathematical understanding look like? One hallmark of mathematical understanding is the ability to justify, in a way appropriate to the student's mathematical maturity, why a particular mathematical statement is accurate or where a mathematical rule comes from. Correctly using language to articulate mathematical understanding plays a part in this justification. Making the distinction between mathematical understanding and procedural skill is critical when designing curriculum and assessment; both are important for the mastery of these standards. That is, there is a world of difference between a student who can summon a mnemonic device to expand a product such as $(a + b)(x + y)$ and a student who can explain what the mnemonic represents as a process for systematically approaching algebraic problems. The student who can explain the rule understands the mathematics, and may have a better chance to succeed at a less familiar task, such as expanding $(a + b + c)(x + y)$.

The Standards set grade-specific standards but do not define the intervention methods or materials necessary to support students who are well below or well above grade-level expectations. It is also beyond the scope of the Standards to define the full range of supports appropriate for English Language Learners (ELLs)/Multilingual Learners (MLLs) and for Students with Disabilities. However, the department ensured that teachers of English Language Learners (ELLs)/Multilingual Learners (MLLs) and Students with Disabilities participated in the revision of the standards. The New York State Education Department (NYSED) has created two statewide frameworks, the *Blueprint for Improved Results for Students with Disabilities* and the *Blueprint for English Language Learner Success*, aimed to clarify expectations and to provide guidance for administrators, policymakers, and practitioners to prepare ELLs/MLLs and Students with Disabilities for success. These principles therein the frameworks are intended to enhance programming and improve instruction that would allow for students within these populations to reach the same standards as all students and leave school prepared to successfully transition to post school learning, living and working.

No set of grade-specific standards can fully reflect the variation in learning profiles, rates, and needs, linguistic backgrounds, and achievement levels of students in any given classroom. When designing and delivering mathematics instruction, educators must consider the cultural context and prior academic experiences of all students while bridging prior knowledge to new knowledge and ensuring that content is meaningful and comprehensible. In addition, as discussed above, educators must consider the relationship of language and content, and the vital role that language plays in obtaining and expressing mathematics content knowledge. The standards should be read as allowing for the widest possible range of students to participate fully from the outset, along with appropriate adaptations to ensure equitable access and maximum participation of all students.

How to Read the P-8 Standards for Mathematical Content

**See High School – Introduction for how to read the High School Standards for Mathematical Content.*

The standards are organized by grade level from Prekindergarten through grade eight.

Standards define what students should understand and be able to do.

Clusters summarize groups of related standards. Note that standards from different clusters may sometimes be closely related, because mathematics is a connected subject.

Domains are larger groups of related standards. Standards from different domains may sometimes be closely related.

Coherence Linkages connect standards one grade level forward and/or back when there are very direct linking standards in those grades. For a more thorough analysis of how standards link to one another, see http://achievethecore.org/coherence-map/.

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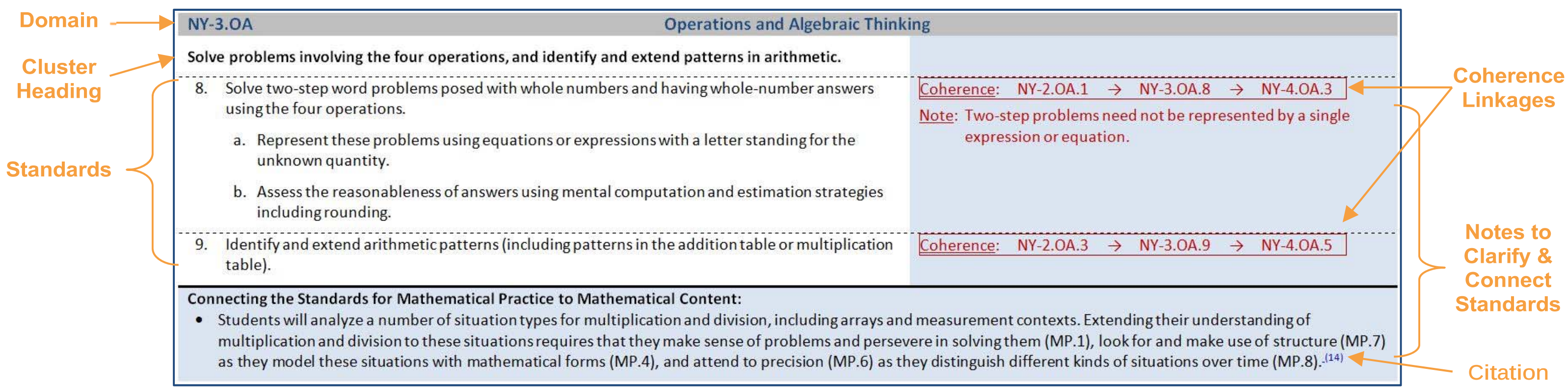
Prekindergarten through Grade Eight



The order in which the standards are presented is not necessarily the order in which the standards need to be taught. Standards from various domains are connected, and educators will need to determine the best overall design and approach, as well as the instructional strategies needed to support their learners to attain grade-level expectations and the knowledge articulated in the standards. That is, the standards do not dictate curriculum or teaching methods; learning opportunities and pathways will continue to vary across schools and school systems, and educators should make every effort to meet the needs of individual students, based on their pedagogical and professional impressions and information.

Pre-Kindergarten Overview

In Pre-Kindergarten, instructional time should focus on two areas: (1) developing a good sense of numbers using concrete objects including concepts of correspondence, counting, cardinality, and comparison; (2) describing shapes in their everyday environment. More learning time in Pre-Kindergarten should be devoted to exploring* and developing the sense of numbers than any other topic. Please note that while every standard/topic in the grade level has not been included in this overview, all standards should be included in instruction.

1. Through their learning in the ***Counting and Cardinality*** domain, students:
   - develop a sense of numbers and count to determine the number of objects;
   - understand that number words refer to quantity;
   - use 1:1 correspondence to solve problems by matching sets and comparing number amounts and in counting objects to 10 through a variety of experiences; and
   - understand that the last number name said tells the number of objects counted (cardinality) and they count to determine number amounts and compare quantities (using language such as more than, fewer than, or equal to (the same as) the number of objects in another group).
2. Through their learning in the ***Geometry*** and ***Measurement and Data*** domains, students:
   - describe the position of objects in space based on the relations of those objects (e.g., shape and special relations) using appropriate vocabulary;
   - identify and name basic two-dimensional shapes, such as triangles, rectangles, squares, and circles; and
   - use basic shapes and spatial reasoning to model objects in their everyday environment.

*Note: *Explore* indicates that the topic is an important concept that builds the foundation for progression toward mastery in later grades. Repeated experiences with these concepts, with immersion in the concrete, are vital.

Mathematical Practices

1. Make sense of problems and persevere in solving them.	5. Use appropriate tools strategically.
2. Reason abstractly and quantitatively.	6. Attend to precision.
3. Construct viable arguments and critique the reasoning of others.	7. Look for and make use of structure.
4. Model with mathematics.	8. Look for and express regularity in repeated reasoning.

NY-PK.CC	Counting and Cardinality
Know number names and the count sequence.	
1. Count to 20.	Coherence: NY-PK.CC.1 → NY-K.CC.1
2. Represent a number of objects (0 - 5), with a written numeral 0–5 (with 0 representing a count of no objects).	Coherence: NY-PK.CC.2 → NY-K.CC.3 Note: Students can select the corresponding number card and/or write the numeral.

Note on Number Reversals:

- Learning to write numerals is generally more difficult than learning to read them. It is common for students to reverse numbers at this stage (e.g., writing Ɛ for 3).[8]

NY-PK.CC	Counting and Cardinality
Count to tell the number of objects.	
3. Understand the relationship between numbers and quantities to 10; connect counting to cardinality. a. When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object. (1:1 correspondence) b. Explore and develop the concept that the last number name said tells the number of objects counted, (cardinality). The number of objects is the same regardless of their arrangement or the order in which they were counted.	Coherence: NY-PK.CC.3 → NY-K.CC.4
4a. Answer counting questions using as many as 10 objects arranged in a line, a rectangular array, and a circle. Answer counting questions using as many as 5 objects in a scattered configuration. 4b. Given a number from 1–10, count out that many objects.	Coherence: NY-PK.CC.4 → NY-K.CC.5 e.g., "How many ______ are there?"

Note on the Word *Explore*:

- *Explore* indicates that the topic is an important concept that builds the foundation for progression toward mastery in later grades. Repeated experiences with these concepts, with immersion in the concrete, are vital.

Within-Grade Connections:

- Much of the learning in prekindergarten—NY-PK.CC.5, all of NY-PK.OA.1, and NY-PK.MD.2—depends on the foundational ability to count to answer "how many?" (NY-PK.CC.4), which itself is grounded in NY-PK.CC.3. Therefore, work on NY-PK.CC.3 & 4 should likely begin at or near the beginning of the year.[9]

NY-PK.CC	Counting and Cardinality
Compare numbers.	
5. Recognize whether the number of objects in one group is more than, fewer than, or equal to (the same as) the number of objects in another group. Note: Include groups with up to five objects.	Coherence: NY-PK.CC.5 → NY-K.CC.6 e.g., using matching and counting strategies
6. Identify "first" and "last" related to order or position.	Coherence: NY-PK.CC.6 → NY-K.CC.4d

NY-PK.OA | **Operations and Algebraic Thinking**

Understand addition as adding to and understand subtraction as taking from.

1. Explore addition and subtraction by using objects, fingers, and responding to real world situations.

In the chart to the right, *Pre-Kindergarten students explore the two unshaded (white) subtypes for* ***Add To*** *and* ***Take From*** *situations.*

All four unshaded (white) subtypes are expectations in Kindergarten.

Grade 1 and 2 students work with all subtypes. Darker shading indicates the four difficult subtypes that students should work with in Grade 1 but need not master until Grade 2.

Coherence: NY-PK.OA.1 → NY-K.OA.1

e.g., If we have 3 apples and add two more, how many apples do we have all together?

Common Addition and Subtraction Situations

	Result Unknown	Change Unknown	Start Unknown
Add To	*A* bunnies sat on the grass. *B* more bunnies hopped there. How many bunnies are on the grass now? $A + B = \square$	*A* bunnies were on the grass. Some more bunnies hopped there. Then there were *C* bunnies. How many bunnies hopped over to the first *A* bunnies? $A + \square = C$	Some bunnies were sitting on the grass. *B* more bunnies hopped there. Then there were *C* bunnies. How many bunnies were on the grass before? $\square + B = C$
Take From	*C* apples were on the table. I ate *B* apples. How many apples are on the table now? $C - B = \square$	*C* apples were on the table. I ate some apples. Then there were *A* apples. How many apples did I eat? $C - \square = A$	Some apples were on the table. I ate *B* apples. Then there were *A* apples. How many apples were on the table before? $\square - B = A$
	Total Unknown	**Both Addends Unknown**	**Addend Unknown**
Put Together/ Take Apart	*A* red apples and *B* green apples are on the table. How many apples are on the table? $A + B = \square$	Grandma has *C* flowers. How many can she put in her red vase and how many in her blue vase? $C = \square + \square$	*C* apples are on the table. *A* are red and the rest are green. How many apples are green? $A + \square = C$ $C - A = \square$
	Difference Unknown	**Bigger Unknown**	**Smaller Unknown**
Compare	*"How many more?" version:* Lucy has *A* apples. Julie has *C* apples. How many more apples does Julie have than Lucy? *"How many fewer?" version:* Lucy has *A* apples. Julie has *C* apples. How many fewer apples does Lucy have than Julie? $A + \square = C$ $C - A = \square$	*Version with "More":* Julie has *B* more apples than Lucy. Lucy has *A* apples. How many apples does Julie have? *Version with "Fewer":* Lucy has *B* fewer apples than Julie. Lucy has *A* apples. How many apples does Julie have? $A + B = \square$	*Version with "More":* Julie has *B* more apples than Lucy. Julie has *C* apples. How many apples does Lucy have? *Version with "Fewer":* Lucy has *B* fewer apples than Julie. Julie has *C* apples. How many apples does Lucy have? $C - B = \square$ $\square + B = C$

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Connecting the Standards for Mathematical Practice to Mathematical Content:

- When students progress from drawing realistic (artistic) pictures of situations to diagramming addition and subtraction situations using circles or other symbols, and making connections between them, they are relating the concrete to the abstract (MP.2) and making their first mathematical models (MP.4).[9]
- A student choosing to use objects, fingers, or a math drawing to represent and solve a word problem is an example of the student using an appropriate tool strategically (MP.5).[9]

NY-PK.OA	Operations and Algebraic Thinking
Understand simple patterns. 2. Duplicate and extend simple patterns using concrete objects.	Coherence: NY-PK.OA.2 → NY-K.OA.6 e.g., What comes next?

Connecting the Standards for Mathematical Practice to Mathematical Content:

- When students duplicate and extend patterns (NY-PK.OA.2), they are noticing regularity and repeated reasoning (MP.8).

NY-PK.MD	Measurement and Data
Describe and compare measurable attributes. 1. Identify measurable attributes of objects, such as length or weight, and describe them using appropriate vocabulary.	Coherence: NY-PK.MD.1 → NY-K.MD.1 e.g., small, big, short, tall, empty, full, heavy, and light

NY-PK.MD	Measurement and Data
Sort objects and count the number of objects in each category. 2. Sort objects and shapes into categories; count the objects in each category. Note: Limit category counts to be less than or equal to 10.	Coherence: NY-PK.MD.2 → NY-K.MD.3

Within-Grade Connections:

- Sorting objects into categories and counting them (NY-PK.MD.2) offers a context for cardinal counting (NY-PK.CC.4) and for comparing numbers (NY-PK.CC.5).[9]

NY-PK.G	Geometry
Identify and describe shapes (squares, circles, triangles, and rectangles).	
1. Describe objects in the environment using names of shapes, and describe the relative positions of these objects using terms such as top, bottom, up, down, above, below, in front of, behind, over, under, and next to.	Coherence: NY-PK.G.1 → NY-K.G.1
2. Name shapes regardless of size.	Coherence: NY-PK.G.2 → NY-K.G.2

NY-PK.G	Geometry
Explore and create two- and three-dimensional objects.	
3. Explore two- and three-dimensional objects and use informal language to describe their similarities, differences, and other attributes.	Coherence: NY-PK.G.3 → NY-K.G.4
4. Create and build shapes from components.	Coherence: NY-PK.G.4 → NY-K.G.5 e.g., sticks and clay balls

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