Math Problem-Solving: Combining Cognitive & Metacognitive Strategies

Grades: 3-12

Solving an advanced math problem independently requires the coordination of a number of complex skills. The student must have the capacity to reliably implement the specific steps of a particular problem-solving process, or cognitive strategy. At least as important, though, is that the student must also possess the necessary metacognitive skills to analyze the problem, select an appropriate strategy to solve that problem from an array of possible alternatives, and monitor the problem-solving process to ensure that it is carried out correctly.

The following strategies combine both cognitive and metacognitive elements (Montague, 1992; Montague & Dietz, 2009). First, the student is taught a 7-step process for attacking a math word problem (cognitive strategy). Second, the instructor trains the student to use a three-part self-coaching routine for each of the seven problem-solving steps (metacognitive strategy).

In the cognitive part of this multi-strategy intervention, the student learns an explicit series of steps to analyze and solve a math problem. Those steps include:

1. **Reading the problem.** The student reads the problem carefully, noting and attempting to clear up any areas of uncertainly or confusion (e.g., unknown vocabulary terms).
2. **Paraphrasing the problem.** The student restates the problem in his or her own words.
3. **'Drawing' the problem.** The student creates a drawing of the problem, creating a visual representation of the word problem.
4. **Creating a plan to solve the problem.** The student decides on the best way to solve the problem and develops a plan to do so.
5. **Predicting/Estimating the answer.** The student estimates or predicts what the answer to the problem will be. The student may compute a quick approximation of the answer, using rounding or other shortcuts.
6. **Computing the answer.** The student follows the plan developed earlier to compute the answer to the problem.
7. **Checking the answer.** The student methodically checks the calculations for each step of the problem. The student also compares the actual answer to the estimated answer calculated in a previous step to ensure that there is general agreement between the two values.

The metacognitive component of the intervention is a three-part routine that follows a sequence of 'Say', 'Ask, 'Check'. For each of the 7 problem-solving steps reviewed above:

- The student first self-instructs by stating, or 'saying', the purpose of the step ('**Say**').
- The student next self-questions by 'asking' what he or she intends to do to complete the step ('**Ask**').
- The student concludes the step by self-monitoring, or 'checking', the successful completion of the step ('**Check**').

While the Say-Ask-Check sequence is repeated across all 7 problem-solving steps, the actual content of the student self-coaching comments changes across the steps.

Table 1 (as well as the attachment at the bottom of the page) shows how each of the steps in the word problem cognitive strategy is matched to the three-part Say-Ask-Check sequence:

'Say-Ask-Check' Metacognitive Prompts Tied to a Word-Problem Cognitive Strategy (Montague, 1992)		
Cognitive Strategy Step	Metacognitive 'Say-Ask-Check' Prompt Targets	Sample Metacognitive 'Say-Ask-Check' Prompts
1. **Read the problem.**	**'Say' (Self-Instruction) Target:** *The student reads and studies the problem carefully before proceeding.* ***'Ask' (Self-Question) Target:*** *Does the student fully understand the problem* ***'Check' (Self-Monitor) Target:*** *Proceed only if the problem is understood.*	**Say:** "I will read the problem. I will reread the problem if I don't understand it." **Ask: "**Now that I have read the problem, do I fully understand it" **Check:** "I understand the problem and will move forward."
2. **Paraphrase the problem.**	**'Say' (Self-Instruction) Target:** *The student restates the problem in order to demonstrate understanding.* ***'Ask' (Self-Question) Target:*** *Is the student able to paraphrase the problem* ***'Check' (Self-Monitor) Target:*** *Ensure that any highlighted key words are relevant to the question.*	**Say:** "I will highlight key words and phrases that relate to the problem question." "I will restate the problem in my own words." **Ask: "**Did I highlight the most important words or phrases in the problem" **Check:** "I found the key words or phrases that will help to solve the problem."
3. **'Draw' the problem.**	**'Say' (Self-Instruction) Target:** *The student creates a drawing of the problem to consolidate understanding.* ***'Ask' (Self-Question) Target:*** *Is there a match between the drawing and the problem* ***'Check' (Self-Monitor) Target:*** *The drawing includes in visual form the key elements of the math problem.*	**Say:** "I will draw a diagram of the problem." **Ask: "**Does my drawing represent the problem" **Check:** "The drawing contains the essential parts of the problem."
4. **Create a plan to solve the problem.**	**'Say' (Self-Instruction) Target:** *The student generates a plan to solve the problem.* ***'Ask' (Self-Question) Target:*** *What plan will help the student to solve this problem* ***'Check' (Self-Monitor) Target:*** *The plan is appropriate to solve the problem.*	**Say:** "I will make a plan to solve the problem." **Ask:** "What is the first step of this plan What is the next step of the plan" **Check:** "My plan has the right steps to solve the problem."
5. **Predict/ estimate the Answer.**	**'Say' (Self-Instruction) Target:** *The student uses estimation or other strategies to predict or estimate the answer.* ***'Ask' (Self-Question) Target:*** *What estimating technique will the student use to predict the answer* ***'Check' (Self-Monitor) Target:*** *The predicted/estimated answer used all of the essential problem information.*	**Say:** "I will estimate what the answer will be." **Ask:** "What numbers in the problem should be used in my estimation" **Check:** "I did not skip any important information in my estimation."
6. **Compute the answer.**	**'Say' (Self-Instruction) Target:** *The student follows the plan to compute the solution to the problem.* ***'Ask' (Self-Question) Target:*** *Does the answer agree with the estimate* ***'Check' (Self-Monitor) Target:*** *The steps in the plan were followed and the operations completed in the correct order.*	**Say:** "I will compute the answer to the problem." **Ask:** "Does my answer sound right" "Is my answer close to my estimate" **Check:** "I carried out all of the operations in the correct order to solve this problem."
7. **Check the answer.**	**'Say' (Self-Instruction) Target:** *The student reviews the computation steps to verify the answer.* ***'Ask' (Self-Question) Target:*** *Did the student check all the steps in solving the problem and are all computations correct* ***'Check' (Self-Monitor) Target:*** *The problem solution appears to have been done correctly.*	**Say:** "I will check the steps of my answer." **Ask:** "Did I go through each step in my answer and check my work" **Check:** ""

Students will benefit from close teacher support when learning to combine the 7-step cognitive strategy to attack math word problems with the iterative 3-step metacognitive Say-Ask-Check sequence. Teachers can increase the likelihood that the student will successfully acquire these skills by using research-supported instructional practices (Burns, VanDerHeyden, & Boice, 2008), including:

- Verifying that the student has the necessary foundation skills to solve math word problems
- Using explicit instruction techniques to teach the cognitive and metacognitive strategies
- Ensuring that all instructional tasks allow the student to experience an adequate rate of success
- Providing regular opportunities for the student to be engaged in active accurate academic responding
- Offering frequent performance feedback to motivate the student and shape his or her learning.

Attachments

- Say-Ask-Check Student Self-Coaching (Metacognitive) Prompts [2]

References

- Burns, M. K., VanDerHeyden, A. M., & Boice, C. H. (2008). Best practices in intensive academic interventions. In A. Thomas & J. Grimes (Eds.), Best practices in school psychology V (pp.1151-1162). Bethesda, MD: National Association of School Psychologists.
- Montague, M. (1992). The effects of cognitive and metacognitive strategy instruction on the mathematical problem solving of middle school students with learning disabilities. Journal of Learning Disabilities, 25, 230-248.
- Montague, M., & Dietz, S. (2009). Evaluating the evidence base for cognitive strategy instruction and mathematical problem solving. Exceptional Children, 75, 285-302.