AP* Calculus Review

The Fundamental Theorems of Calculus

Teacher Packet

The Fundamental Theorems of Calculus

I. If f is continuous on [a, b], then the function $F(x) = \int_a^x f(t)\,dt$ has a derivative at every point in [a, b], and $\frac{dF}{dx} = \frac{d}{dx}\int_a^x f(t)\,dt = f(x)$.

II. If f is continuous on [a, b], and if F is any antiderivative of f on [a, b], then $\int_a^b f(t)\,dt = F(b) - F(a)$.

Note: These two theorems may be presented in reverse order. Part II is sometimes called the Integral Evaluation Theorem.

Don't overlook the obvious!

1. $\frac{d}{dx}\int_a^a f(t)\,dt = 0$, because the definite integral is a constant
2. $\int_a^b f'(x)\,dx = f(b) - f(a)$

Upgrade for part I, applying the Chain Rule

If $F(x) = \int_a^{g(x)} f(t)\,dt$, then $\frac{dF}{dx} = \frac{d}{dx}\int_a^{g(x)} f(t)\,dt = f(g(x))g'(x)$.

For example, $\frac{d}{dx}\int_2^{x^3} \sin(t^2)\,dt = \left(\left(\sin(x^3)^2\right)\right)\left(3x^2\right) = 3x^2\sin\left(x^6\right)$

An important alternate form for part II

$F(b) = F(a) + \int_a^b f(t)\,dt$

[Think of this as: ending value = starting value plus accumulation.]

For example, given that $\int_3^{12} f'(x)\,dx = -4$ and $f(3) = 35$, find $f(12)$.

Using the alternate format, $f(12) = f(3) + \int_3^{12} f'(x)\,dx = 35 + (-4) = 31$.

Sample Problems

Multiple Choice – No Calculator

1. $\frac{d}{dx}\int_{2}^{x} \ln t \, dt =$

(A) $\ln x$ (B) $\ln 2$ (C) $\frac{1}{x}$

(D) $\frac{1}{2}$ (E) $\ln x - \ln 2$

2. If $g(x) = \int_{\pi}^{\pi x} \cos(t^2)\, dt$, then $g'(x) =$

(A) $\sin\left(\pi^2 x^2\right)$ (B) $\pi x \sin\left(\pi^2 x^2\right)$ (C) $\pi x \cos\left(\pi^2 x^2\right)$

(D) $\cos\left(\pi^2 x^2\right)$ (E) $\pi \cos\left(\pi^2 x^2\right)$

3. $\frac{d}{dx}\int_{\sin x}^{4} \sqrt{1+t^2}\, dt =$

(A) $\sqrt{1+\sin^2 x}$ (B) $-\cos x\sqrt{1+\sin^2 x}$ (C) $-\sqrt{1+\sin^2 x}$

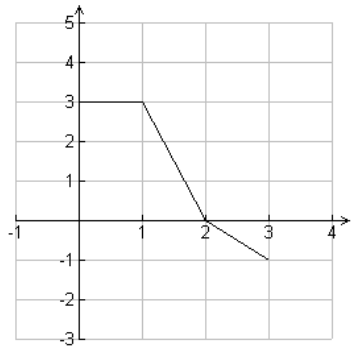
(D) $\cos x\sqrt{1+\sin^2 x}$ (E) $\sqrt{1+\cos^2 x}$

4. If f has two continuous derivatives on [5, 10], then $\int_{5}^{10} f''(t)\, dt =$

(A) $f'''(10) - f'''(5)$ (B) $f(10) - f(5)$ (C) $f'(10) - f'(5)$

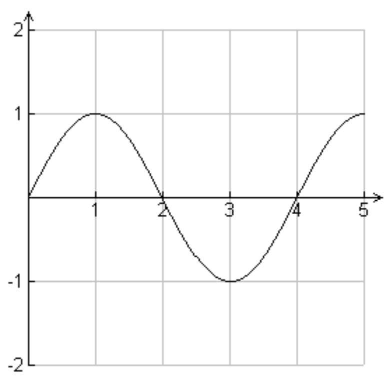
(D) $f''(10) - f''(5)$ (E) $f''(5) - f''(10)$

5. The graph of f is given, and g is an antiderivative of f. If $g(3) = 6$, find $g(0)$.



(A) 1 (B) 2 (C) 4 (D) 5 (E) 10

6. The graph of f is given. $F(x) = \int_0^x f(t)\,dt$



Which of the following statements is true?
(A) F decreases on (1, 2).
(B) F has a relative minimum at $x = 2$
(C) F decreases on (2, 4)
(D) F has a relative maximum at $x = 1$.
(E) F has a point of inflection at $x = 4$.

7. $\dfrac{d}{dx}\displaystyle\int_{x}^{x^2} \tan(t)\,dt =$

(A) $\tan\left(x^2\right) - \tan x$ (B) $\tan x - \tan\left(x^2\right)$

(C) $\tan x - 2x\tan\left(x^2\right)$ (D) $2x\tan\left(x^2\right) - \tan x$

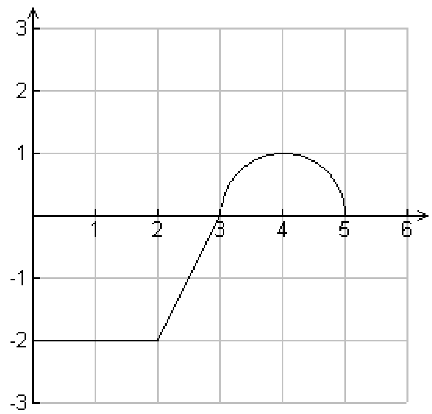
(E) $\sec^2\left(x^2\right) - \sec^2 x$

8. $\displaystyle\int_{1}^{e} \left(x - \frac{5}{x}\right) dx =$

(A) $\frac{1}{2}e^2 - \frac{11}{2}$ (B) $\frac{1}{2}e^2 - \frac{9}{2}$ (C) $e^2 - \frac{11}{2}$

(D) $\frac{1}{2}e^2 - \frac{3}{2}$ (E) $\frac{11}{2} - \frac{1}{2}e^2$

Free Response 1 – No Calculator



The graph of f is given. It consists of two line segments and a semi-circle.
$g(x) = \int_1^x f(t)\,dt$

(a) Find $g(0)$, $g(1)$, and $g(5)$.

(b) Find $g'(2)$, $g''(2)$, and $g'''(4)$ or state that it does not exist.

(c) For what value(s) of x does the graph of g have a point of inflection? Justify your answer.

(d) Find the absolute maximum and absolute minimum values of g on [0, 5]. Justify your answer.

Multiple Choice – Calculator Allowed

1. If $g(x)=\int_0^x \sin^2 t\, dt$, then $g'(2)=$

   (A) 0 (B) 0.001 (C) 0 .173 (D) 0 .827 (E) 1.189

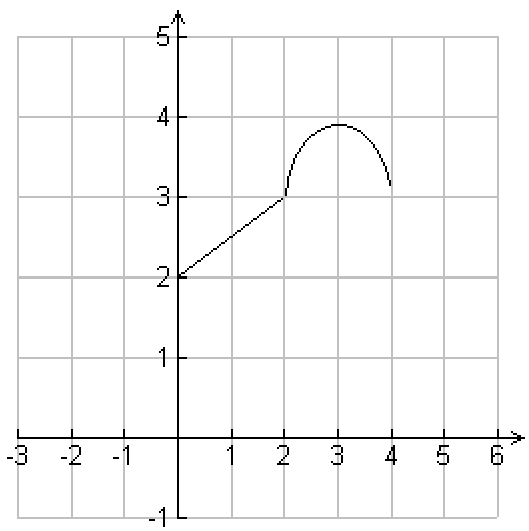
2. A car sold for $16,000 and depreciated at a rate of $2e^{x^2}$ dollars per year. What is the value of the car 3 years after the purchase?

   (A) $206.17 (B) $2889.09 (C) $13,110.91

   (D) $16,206.17 (E) $18,889.09

3. The graph of f is given, and *F(x)* is an antiderivative of *f*. If $\int_2^4 f(x)\,dx = 7.5$, find $F(4)-F(0)$.

   (A) 1 (B) 1.5 (C) 7.5 (D) 12.5 (E) 18.5



4. The acceleration of an object in motion is defined by $\sqrt{1+t^2}$. The velocity at $t = 6$ is 22. Find the velocity at $t = 1$.

(A) 1.414 (B) 3.654 (C) 18.346 (D) 22.023 (E) 30.346

5. $h(x) = \int_1^x g(t)\,dt$ and $g(t) = \int_0^{t^2} \frac{\sqrt{1+u^2}}{u}\,du$. Find $h''(2.5)$.

(A) 1.013 (B) 1.077 (C) 2.154 (D) 5.064 (E) 12.659

6. Find $\int_{-2}^{2} f(x)\,dx$ if $f(x) = \begin{cases} 2x^2, & -2 \le x \le 0 \\ \sin 2x, & 0 < x \le 2 \end{cases}$

(A) 0 (B) 4.507 (C) 5.403 (D) 6.161 (E) 10.667

7. Let $g(x)$ be an antiderivative of $\frac{x^3}{\ln x}$. If $g(2) = 3$, find $g(6)$.

(A) 120.552 (B) 123.552 (C) 208.122

(D) 211.122 (E) 214.122

8. $h(x) = \int_0^{2x} (e^{\cos t} - 1)\,dt$ on (3, 6). On which interval(s) is h decreasing?

(A) (3.927, 5.498) (B) (5.498, 6)

(C) (3, 4.712) (D) Always decreasing on (3, 6)

(E) Never decreasing on (3, 6)

Free Response – Calculator Active

Let $g(x)=\int_{1}^{x}(5-8\sqrt{\ln t})\,dt$ for $x > 1$. Let $h(x)=\int_{1}^{x^2}(5-8\sqrt{\ln t})\,dt$ for $x > 1$.

(a) Write an equation of the tangent to g at $x = 3$.

(b) What is $h'(x)$?

(c) On which open interval(s) is g decreasing? Justify your answer?

(d) Find all x values for which h has relative extrema. Label them as maximum or minimum and justify your answer.