AP Calculus AB

Review Week 2

Derivatives and Tangents

Advanced Placement AAP Review will be held in **room 315** and **312** on Tuesdays and Thursdays.

The week of March 23rd we will be reviewing **Derivatives and Tangents**.

The session will begin in room 315 with a brief review of the weekly topic.

Instruction will be from 3:00 pm to 3:15 pm

Once we have reviewed the topic you may begin practicing the questions in your review packet.

Answers will be posted in room 315 and 312 all week and will be posted on line after 3:00 pm on Friday the week of review.

If you have difficulty with a question look at the detailed answer postings BEFORE you ask your teacher for help.

Get a hint….**DON'T COPY THE ANSWER!!! THAT IS NOT HELPFUL!!**

When you have completed a question…**REFLECT!!!!** Ask yourself what skill you used to solve that problem and write that down!!

Once we have completed the weekly review, keep it to study from as we get closer to the exam.

Derivatives and Tangents

Brief Review

Derivative – Slope of the tangent line. (Instantaneous rate of change – at ONE point)

Average rate of change is ordinary old slope. (slope of the secant line actually, from TWO points)

Techniques:

- POWER RULE: this is the one you like…Multiple by the exponent…decrease the exponent by one.

  Power rule works for all exponents…positive, negative, rational…etc.

- PRODUCT RULE: First D(second) + Second D(first)
- QUOTIENT RULE: Low D(high) – high D(low) ÷ $(\text{low})^2$
- CHAIN RULE: D(outside) times D(inside)…you always forget this…DON'T!!!!
- D$(e^x) = e^x$ ….this is THE BEST RULE EVER.
- D$(lnx) = \frac{1}{x}$
- You can take the derivative of piecewise function…take the derivative of each piece.
- Know the derivative of the sine, cosine and tangent.
- You should be able to perform IMPLICIT DIFFERENTIATION!!!....that is when you are not solved for y explicitly.
- You should know the derivatives of your INVERSE sine, cosine and tangent.
- Little used so forgotten….D$(a^x) = a^x lna$….
- BC…Logarithmic Differentiation.

KNOW:

$f(x) position$

$f'(x) velocity$ speed is the absolute value of velocity

$f''(x) acceleration$

NORMAL LINE is perpendicular to the tangent line.

Linearization means tangent line…linear approximation means tangent line…

Slope of the curve…derivative…gradient of the curve…derivative…rate of change…derivative…

Slope at a point…derivative…slope of the graph…derivative…slope of the tangent line…derivative

2008 Non Calculator Active

3. If $f(x)=(x-1)(x^2+2)^3$, then $f'(x)=$

(A) $6x(x^2+2)^2$

(B) $6x(x-1)(x^2+2)^2$

(C) $(x^2+2)^2(x^2+3x-1)$

(D) $(x^2+2)^2(7x^2-6x+2)$

(E) $-3(x-1)(x^2+2)^2$

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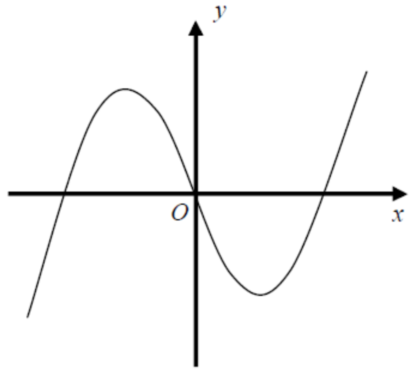
8. If $f(x)=\cos(3x)$, then $f'\left(\frac{\pi}{9}\right)=$

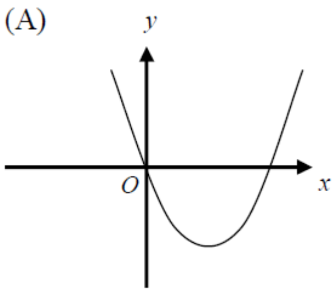
(A) $\frac{3\sqrt{3}}{2}$ (B) $\frac{\sqrt{3}}{2}$ (C) $-\frac{\sqrt{3}}{2}$ (D) $-\frac{3}{2}$ (E) $-\frac{3\sqrt{3}}{2}$



Graph of f

11. The graph of a function f is shown above. Which of the following could be the graph of f', the derivative of f?

(A)



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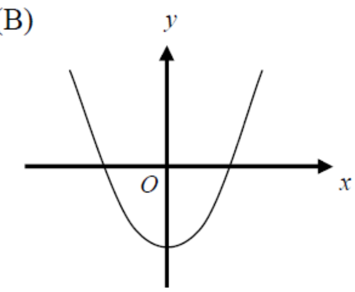

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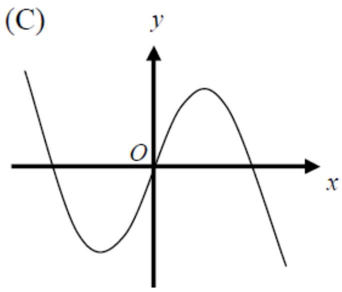

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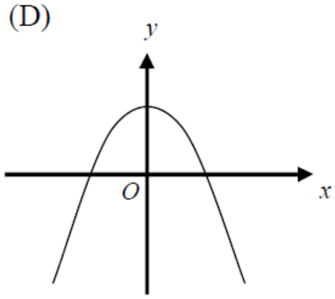
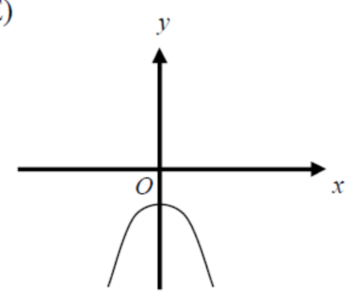


(E)



12. If $f(x) = e^{(2/x)}$, then $f'(x) =$

(A) $2e^{(2/x)} \ln x$ (B) $e^{(2/x)}$ (C) $e^{(-2/x^2)}$ (D) $-\frac{2}{x^2}e^{(2/x)}$ (E) $-2x^2e^{(2/x)}$

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13. If $f(x) = x^2 + 2x$, then $\frac{d}{dx}(f(\ln x)) =$

(A) $\frac{2\ln x + 2}{x}$ (B) $2x\ln x + 2$ (C) $2\ln x + 2$ (D) $2\ln x + \frac{2}{x}$ (E) $\frac{2x+2}{x}$

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18. In the xy-plane, the line $x + y = k$, where k is a constant, is tangent to the graph of $y = x^2 + 3x + 1$. What is the value of k?

(A) -3 (B) -2 (C) -1 (D) 0 (E) 1

16. If $\sin(xy) = x$, then $\frac{dy}{dx} =$

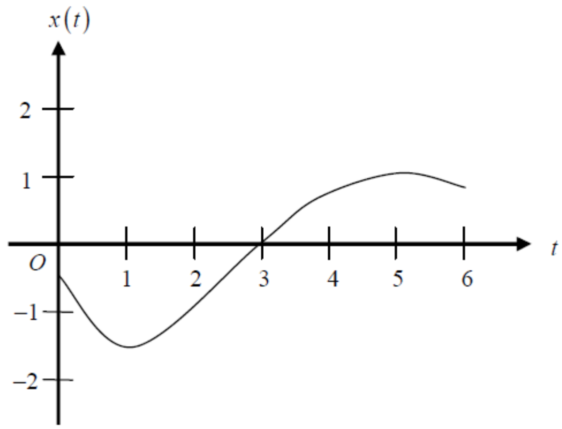
(A) $\frac{1}{\cos(xy)}$

(B) $\frac{1}{x\cos(xy)}$

(C) $\frac{1-\cos(xy)}{\cos(xy)}$

(D) $\frac{1-y\cos(xy)}{x\cos(xy)}$

(E) $\frac{y(1-\cos(xy))}{x}$



21. A particle moves along a straight line. The graph of the particle's position $x(t)$ at time t is shown above for $0 < t < 6$. The graph has horizontal tangents at $t = 1$ and $t = 5$ and a point of inflection at $t = 2$. For what values of t is the velocity of the particle increasing?

(A) $0 < t < 2$

(B) $1 < t < 5$

(C) $2 < t < 6$

(D) $3 < t < 5$ only

(E) $1 < t < 2$ and $5 < t < 6$

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24. The function f is twice differentiable with $f(2) = 1$, $f'(2) = 4$, and $f''(2) = 3$. What is the value of the approximation of $f(1.9)$ using the line tangent to the graph of f at $x = 2$?

(A) 0.4 (B) 0.6 (C) 0.7 (D) 1.3 (E) 1.4

$$f(x)=\begin{cases} cx+d & \text{for } x\le 2 \\ x^2-cx & \text{for } x>2 \end{cases}$$

25. Let f be the function defined above, where c and d are constants. If f is differentiable at $x=2$, what is the value of $c+d$?

(A) -4 (B) -2 (C) 0 (D) 2 (E) 4

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26. What is the slope of the line tangent to the curve $y=\arctan(4x)$ at the point at which $x=\frac{1}{4}$?

(A) 2 (B) $\frac{1}{2}$ (C) 0 (D) $-\frac{1}{2}$ (E) -2

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28. Let f be a differentiable function such that $f(3)=15$, $f(6)=3$, $f'(3)=-8$, and $f'(6)=-2$. The function g is differentiable and $g(x)=f^{-1}(x)$ for all x. What is the value of $g'(3)$?

(A) $-\frac{1}{2}$

(B) $-\frac{1}{8}$

(C) $\frac{1}{6}$

(D) $\frac{1}{3}$

(E) The value of $g'(3)$ cannot be determined from the information given.

82. A particle moves along a straight line with velocity given by $v(t) = 7 - (1.01)^{-t^2}$ at time $t \geq 0$. What is the acceleration of the particle at time $t = 3$?

(A) –0.914 (B) 0.055 (C) 5.486 (D) 6.086 (E) 18.087

2003 Non-Calculator Active

1. If $y = (x^3 + 1)^2$, then $\frac{dy}{dx} =$

(A) $(3x^2)^2$ (B) $2(x^3 + 1)$ (C) $2(3x^2 + 1)$ (D) $3x^2(x^3 + 1)$ (E) $6x^2(x^3 + 1)$

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4. If $y = \frac{2x + 3}{3x + 2}$, then $\frac{dy}{dx} =$

(A) $\frac{12x + 13}{(3x + 2)^2}$ (B) $\frac{12x - 13}{(3x + 2)^2}$ (C) $\frac{5}{(3x + 2)^2}$ (D) $\frac{-5}{(3x + 2)^2}$ (E) $\frac{2}{3}$

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9. If $f(x) = \ln(x + 4 + e^{-3x})$, then $f'(0)$ is

(A) $-\frac{2}{5}$ (B) $\frac{1}{5}$ (C) $\frac{1}{4}$ (D) $\frac{2}{5}$ (E) nonexistent

14. If $y = x^2 \sin 2x$, then $\frac{dy}{dx} =$

(A) $2x \cos 2x$

(B) $4x \cos 2x$

(C) $2x(\sin 2x + \cos 2x)$

(D) $2x(\sin 2x - x \cos 2x)$

(E) $2x(\sin 2x + x \cos 2x)$

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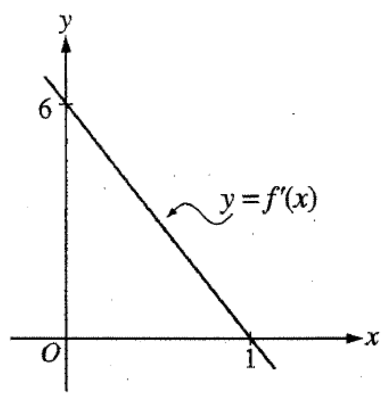
16. If the line tangent to the graph of the function f at the point $(1, 7)$ passes through the point $(-2, -2)$, then $f'(1)$ is

(A) -5 (B) 1 (C) 3 (D) 7 (E) undefined



22. The graph of f', the derivative of f, is the line shown in the figure above. If $f(0) = 5$, then $f(1) =$

(A) 0 (B) 3 (C) 6 (D) 8 (E) 11

24. Let f be the function defined by $f(x) = 4x^3 - 5x + 3$. Which of the following is an equation of the line tangent to the graph of f at the point where $x = -1$?

(A) $y = 7x - 3$

(B) $y = 7x + 7$

(C) $y = 7x + 11$

(D) $y = -5x - 1$

(E) $y = -5x - 5$

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25. A particle moves along the x-axis so that at time $t \geq 0$ its position is given by $x(t) = 2t^3 - 21t^2 + 72t - 53$. At what time t is the particle at rest?

(A) $t = 1$ only

(B) $t = 3$ only

(C) $t = \frac{7}{2}$ only

(D) $t = 3$ and $t = \frac{7}{2}$

(E) $t = 3$ and $t = 4$

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26. What is the slope of the line tangent to the curve $3y^2 - 2x^2 = 6 - 2xy$ at the point $(3, 2)$?

(A) 0 (B) $\frac{4}{9}$ (C) $\frac{7}{9}$ (D) $\frac{6}{7}$ (E) $\frac{5}{3}$

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27. Let f be the function defined by $f(x) = x^3 + x$. If $g(x) = f^{-1}(x)$ and $g(2) = 1$, what is the value of $g'(2)$?

(A) $\frac{1}{13}$ (B) $\frac{1}{4}$ (C) $\frac{7}{4}$ (D) 4 (E) 13

2003 Calculator Active

76. A particle moves along the x-axis so that at any time $t \geq 0$, its velocity is given by $v(t) = 3 + 4.1\cos(0.9t)$. What is the acceleration of the particle at time $t = 4$?

(A) -2.016 (B) -0.677 (C) 1.633 (D) 1.814 (E) 2.978

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89. Let f be a differentiable function with $f(2) = 3$ and $f'(2) = -5$, and let g be the function defined by $g(x) = xf(x)$. Which of the following is an equation of the line tangent to the graph of g at the point where $x = 2$?

(A) $y = 3x$

(B) $y - 3 = -5(x - 2)$

(C) $y - 6 = -5(x - 2)$

(D) $y - 6 = -7(x - 2)$

(E) $y - 6 = -10(x - 2)$

2013 Free Response

Calculator

1. On a certain workday, the rate, in tons per hour, at which unprocessed gravel arrives at a gravel processing plant is modeled by $G(t) = 90 + 45\cos\left(\frac{t^2}{18}\right)$, where t is measured in hours and $0 \le t \le 8$. At the beginning of the workday $(t = 0)$, the plant has 500 tons of unprocessed gravel. During the hours of operation, $0 \le t \le 8$, the plant processes gravel at a constant rate of 100 tons per hour.

   (a) Find $G'(5)$. Using correct units, interpret your answer in the context of the problem.

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2. A particle moves along a straight line. For $0 \le t \le 5$, the velocity of the particle is given by $v(t) = -2 + \left(t^2 + 3t\right)^{6/5} - t^3$, and the position of the particle is given by $s(t)$. It is known that $s(0) = 10$.

   (a) Find all values of t in the interval $2 \le t \le 4$ for which the speed of the particle is 2.

Non-Calculator

t (minutes)	0	1	2	3	4	5	6
$C(t)$ (ounces)	0	5.3	8.8	11.2	12.8	13.8	14.5

3. Hot water is dripping through a coffeemaker, filling a large cup with coffee. The amount of coffee in the cup at time t, $0 \le t \le 6$, is given by a differentiable function C, where t is measured in minutes. Selected values of $C(t)$, measured in ounces, are given in the table above.

   (a) Use the data in the table to approximate $C'(3.5)$. Show the computations that lead to your answer, and indicate units of measure.

6. Consider the differential equation $\frac{dy}{dx} = e^y(3x^2 - 6x)$. Let $y = f(x)$ be the particular solution to the differential equation that passes through $(1, 0)$.

(a) Write an equation for the line tangent to the graph of f at the point $(1, 0)$. Use the tangent line to approximate $f(1.2)$.

2012 Exam

Calculator

t (minutes)	0	4	9	15	20
$W(t)$ (degrees Fahrenheit)	55.0	57.1	61.8	67.9	71.0

1. The temperature of water in a tub at time t is modeled by a strictly increasing, twice-differentiable function W, where $W(t)$ is measured in degrees Fahrenheit and t is measured in minutes. At time $t = 0$, the temperature of the water is 55°F. The water is heated for 30 minutes, beginning at time $t = 0$. Values of $W(t)$ at selected times t for the first 20 minutes are given in the table above.

(a) Use the data in the table to estimate $W'(12)$. Show the computations that lead to your answer. Using correct units, interpret the meaning of your answer in the context of this problem.

Non-Calculator

4. The function f is defined by $f(x) = \sqrt{25 - x^2}$ for $-5 \le x \le 5$.

   (a) Find $f'(x)$.

   (b) Write an equation for the line tangent to the graph of f at $x = -3$.

   (c) Let g be the function defined by $g(x) = \begin{cases} f(x) & \text{for } -5 \le x \le -3 \\ x + 7 & \text{for } -3 < x \le 5. \end{cases}$

   Is g continuous at $x = -3$? Use the definition of continuity to explain your answer.

5. The rate at which a baby bird gains weight is proportional to the difference between its adult weight and its current weight. At time $t = 0$, when the bird is first weighed, its weight is 20 grams. If $B(t)$ is the weight of the bird, in grams, at time t days after it is first weighed, then

$$\frac{dB}{dt} = \frac{1}{5}(100 - B).$$

Let $y = B(t)$ be the solution to the differential equation above with initial condition $B(0) = 20$.

(a) Is the bird gaining weight faster when it weighs 40 grams or when it weighs 70 grams? Explain your reasoning.

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6. For $0 \le t \le 12$, a particle moves along the x-axis. The velocity of the particle at time t is given by $v(t) = \cos\left(\frac{\pi}{6}t\right)$. The particle is at position $x = -2$ at time $t = 0$.

(a) For $0 \le t \le 12$, when is the particle moving to the left?

2011 Exam

Calculator

1. For $0 \le t \le 6$, a particle is moving along the x-axis. The particle's position, $x(t)$, is not explicitly given. The velocity of the particle is given by $v(t) = 2\sin\left(e^{t/4}\right) + 1$. The acceleration of the particle is given by $a(t) = \frac{1}{2}e^{t/4}\cos\left(e^{t/4}\right)$ and $x(0) = 2$.

(a) Is the speed of the particle increasing or decreasing at time $t = 5.5$? Give a reason for your answer.

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t (minutes)	0	2	5	9	10
$H(t)$ (degrees Celsius)	66	60	52	44	43

2. As a pot of tea cools, the temperature of the tea is modeled by a differentiable function H for $0 \le t \le 10$, where time t is measured in minutes and temperature $H(t)$ is measured in degrees Celsius. Values of $H(t)$ at selected values of time t are shown in the table above.

   (a) Use the data in the table to approximate the rate at which the temperature of the tea is changing at time $t = 3.5$. Show the computations that lead to your answer.

Non-Calculator

5. At the beginning of 2010, a landfill contained 1400 tons of solid waste. The increasing function W models the total amount of solid waste stored at the landfill. Planners estimate that W will satisfy the differential equation $\frac{dW}{dt} = \frac{1}{25}(W - 300)$ for the next 20 years. W is measured in tons, and t is measured in years from the start of 2010.

(a) Use the line tangent to the graph of W at $t = 0$ to approximate the amount of solid waste that the landfill contains at the end of the first 3 months of 2010 (time $t = \frac{1}{4}$).

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6. Let f be a function defined by $f(x) = \begin{cases} 1 - 2\sin x & \text{for } x \le 0 \\ e^{-4x} & \text{for } x > 0. \end{cases}$

(b) For $x \neq 0$, express $f'(x)$ as a piecewise-defined function. Find the value of x for which $f'(x) = -3$.