

Chapter 3 Worksheet Packet

AP Calculus AB

Name _____

Calculus Practice: Derivatives

Find the derivative and give the domain of the derivative for each of the following functions. If the derivative does not exist at any point, explain why and justify your answer.

1) $f(x) = x^{\frac{3}{5}} + 2$

2) $f(x) = x^{\frac{4}{7}}$

3) $f(x) = |2x + 1|$

4) $f(x) = \begin{cases} x^3 & \text{if } x \leq 0 \\ 2x & \text{if } x > 0 \end{cases}$

Find the equation of the tangent line to the graph of $f(x)$ at the point P .

5) $f(x) = \frac{2}{x^2}$ $P(1, 2)$

6) $f(x) = x^{\frac{2}{3}} + 2x$ $P(8, 20)$

7) Find the velocity of the particle at time $t = 3$ if the position function for the particle is given by $s(t) = t^3 - 2t$.

8) Find the x values of all points on the graph of $y = x^4 - 2x^2$ where the tangent line is horizontal.

AP Calculus Practice (3.1-3.3)

For each of the following functions, find the derivative. For each derivative, determine all values for which the derivative does not exist. For each of these values determine if the derivative does not exist due to a discontinuity, a corner point, a cusp, or a vertical tangent line.

$$1) f(x) = \frac{3x^{\frac{2}{3}}}{x^3 + 1}$$

$$2) f(x) = 4x^{\frac{3}{4}}(3x^4 - 1)$$

$$3) f(x) = \frac{x^{\frac{1}{5}}}{2x^5 + 3}$$

$$4) f(x) = 2x^{\frac{3}{7}}(5x^3 + 1)$$

$$5) f(x) = \frac{x^3}{x^4 + 1}$$

$$6) f(x) = 3x^3(4x^2 + 3)$$

Find the equation of the tangent line to the graph of the given function at the given point.

$$7) f(x) = \sqrt[3]{x^2} + \frac{3}{x^2} - 2 \text{ at } x = 1$$

$$8) g(x) = \frac{2x}{x^2 + 2} \text{ at } x = 2$$

Find the equation of the normal line to the graph of the given function at the given point.

$$9) f(x) = \sqrt[5]{x^6} - \frac{2}{x^3} - 5 \text{ at } x = -1$$

$$10) g(x) = x^2(3x^4 + 1) \text{ at } x = 1$$

AP Calculus
Derivatives Practice

Name _____

Date _____

1. If $f(x) = \sin^4 x$, then $f'(\frac{\pi}{3}) =$

a) $-\frac{\sqrt{3}}{4}$

b) $9\sqrt{3}$

c) $\frac{27}{4}$

d) $\frac{\sqrt{3}}{4}$

e) $\frac{3\sqrt{3}}{4}$

2. Given $f(x) = \frac{x}{\tan x}$, find $f'(\frac{\pi}{2})$.

a) $\frac{1}{2}$

b) $\frac{\pi}{2}$

c) $\frac{\pi}{2} - 1$

d) 4

e) undefined

3. Differentiate: $f(x) = x^2 + 2 \tan x$

a) $2x + 2 \tan x$

b) $2x + \sec^2 x$

c) $2 + \sec^2 x$

d) $2x + 2 \sec^2 x$

e) $2x + 2 \cot x$

4. Find the derivative, $\frac{dy}{dx}$, of $y = \frac{3x}{x^2 + 1}$.

a) $\frac{3}{1+x^2}$

b) $\frac{3}{2x}$

c) $\frac{3x^2 - 3}{(1+x^2)^3}$

d) $\frac{3(1-x^2)}{(1+x^2)^2}$

e) $\frac{6x+x^2}{(x^2+1)^2}$

5. Find the derivative, $\frac{dy}{dx}$, of $f(x) = \frac{x^2 - 1}{x^2 + 1}$.

a) $\frac{4x}{(x^2 + 1)^2}$

b) 1

c) $-\frac{4x}{(x^2 + 1)^2}$

d) $\frac{4x^2}{(x^2 + 1)^2}$

e) $\frac{-4x^2 - 4x}{(x^2 + 1)^2}$

6. If $f(x) = (x^3 + 4x^2 - 12x + 8)(3x^2 - 9x + 7)$, then find $f'(1)$.

a) -4

b) 4

c) -3

d) 3

e) 7

7. Find the slope of a line tangent to the graph of $f(x) = \frac{x+3}{x+2}$ at the point $(1, \frac{4}{3})$.

a) $-\frac{5}{9}$

b) $-\frac{1}{9}$

c) $\frac{1}{9}$

d) $\frac{5}{9}$

e) $\frac{3}{2}$

8. Find an equation of the tangent line to the curve $f(x) = x^2 - 10$ passing through the point $(5, 1)$.
- a) $y - 1 = -10(x - 5)$ b) $y + 5 = -10(x + 1)$ c) $y + 1 = 10(x + 5)$
d) $y - 1 = 10(x - 5)$ e) $y - 5 = 10(x - 1)$
9. Find the slope of the tangent to the graph $f(x) = \frac{\sin x}{\cos 2x}$ where $x = \frac{\pi}{6}$.
- a) $\frac{\sqrt{3}}{2}$ b) $\frac{2\sqrt{3}}{3}$ c) 3 d) $\sqrt{3}$ e) $3\sqrt{3}$
10. Find $f'(x)$ for $f(x) = (2x^2 + 5)^7$.
- a) $7(4x)^6$ b) $(4x)^7$ c) $28x(2x^2 + 5)^6$ d) $7(2x^2 + 5)^6$ e) $28x^7$
11. Find $\frac{dy}{dx}$ for $y = x^3\sqrt{2x+1}$
- a) $\frac{x^2(7x+3)}{\sqrt{2x+1}}$ b) $\frac{3x^2}{2\sqrt{2x+1}}$ c) $\frac{8x^3+3x^2}{2\sqrt{2x^4+x^3}}$ d) $\frac{8x+3}{\sqrt{2x+1}}$ e) $\frac{6x^3+3}{\sqrt{2x+1}}$
12. If $y = (3x^2 + 5)^5(x + 2)^4$, then $\frac{dy}{dx} =$
- a) $2(x + 2)^3(3x^2 + 5)^4$ b) $2(21x^2 + 30x + 10)(x + 2)^3(3x^2 + 5)^4$
c) $(x + 2)^3(3x^2 + 5)(21x^2 + 30x + 10)$ d) $24(x + 2)^3(3x^2 + 5)^4(21x^2 + 30x + 10)$
e) $12(x + 2)^3(3x^2 + 5)^4(21x + 30)$
13. Find the derivative of $y = \cos x^3$.
- a) $3x^2 \sin x^3$ b) $3 \cos x^3$ c) $-3x^2 \sin x^3$ d) $3 \sin x^3 \cos^2 x^3$ e) $3x \cos x^2$
14. Find $f'(x)$ given $f(x) = \sin^3(4x)$.
- a) $4 \cos^3(4x)$ b) $3 \sin^2 4x \cos(4x)$ c) $\cos^3 4x$
d) $12 \sin^2 4x \cos(4x)$ e) $12 \cos^2(4x)$

5. The ends of a water trough 14 ft. long are equilateral triangles whose sides are 5 ft. long. If the water is being pumped into the trough at a rate of 10 cu. ft. per min., find the rate at which the water level is rising when the depth is 6 *in.*
6. A spherical balloon is being inflated with gas. Use differentials to approximate the increase in surface area of the balloon if the radius changes from 2 ft. to 2.05 ft.
7. Use a linear approximation to estimate the value of $\sqrt[3]{127}$.
8. The radius of a spherical balloon is measured as 8 in. with a maximum error of .1 in. Approximate the relative error for the calculated value of the volume.

Chapter 3 Test Practice/AP Calculus

The equation gives the position $s = f(t)$ of a body moving on a coordinate line (s in meters, t in seconds).

1) $s = 6 \sin t - \cos t$

Find the body's velocity at time $t = \pi/6$ sec.

Find the derivative of the given function.

2) $y = 2 \sin^{-1}(4x^3)$

3) $y = \tan^{-1} \sqrt{3x}$

4) $y = \sqrt{3 + \sin 4x}$

5) $y = \ln 4x^2$

6) $y = \frac{7}{\sin x}$

7) $y = 5 \sec^2 x$

8) $y = 4^x$

9) $y = \cot(2x - 5)$

10) $y = 5 \sec^6 x$

11) $y = \log(5x - 4)$

12) $y = 3xe^x - 3e^x$

Solve.

13) Find the tangent line to the graph of $x^2 + y^2 - 2x + 4y = 8$ at $(4, 0)$

14) Find the normal line to the graph of $4x^2y - \pi \cos y = 5\pi$ at $(1, \pi)$

Use implicit differentiation to find d^2y/dx^2 .

15) $y^2 - x^2 = 9$

Solve the problem.

16) The position of a particle moving along a coordinate line is $s = \sqrt{5 + 4t}$, with s in meters and t in seconds. Find the particle's velocity at $t = 1$ sec.

17) The profit in dollars from the sale of x thousand compact disc players is $P(x) = x^3 - 3x^2 + 4x + 8$. Find the marginal profit when the value of x is 9.

18) At time t , the position of a body moving along the s -axis is $s = t^3 - 21t^2 + 144t$ m. Find the body's acceleration each time the velocity is zero.

Solve the problem.

19) The function $V = \frac{4}{3}\pi r^3$ describes the volume of a sphere of radius r feet. Find the (instantaneous) rate of change of the volume with respect to the radius when $r = 9$. Leave answer in terms of π .

If the function is not differentiable at the given value of x , tell whether the problem is a corner, cusp, vertical tangent, or a discontinuity.

20) $y = 2x - \sqrt[5]{x}$, at $x = 0$

Use logarithmic differentiation to find dy/dx .

21) $y = (5x)^x$

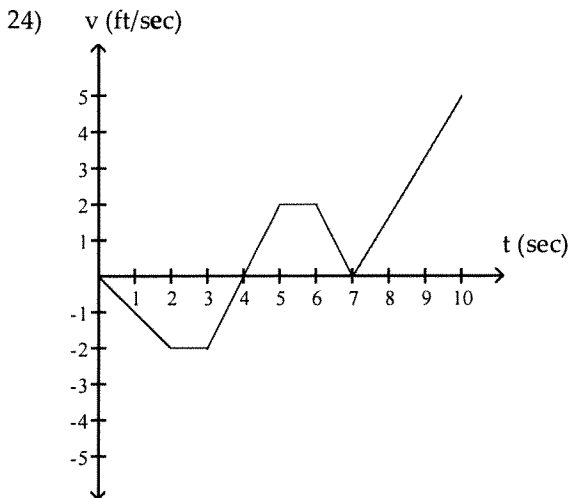
Find an equation for the line tangent to the curve at the point defined by the given value of t .

22) $x = 4 \cos t$, $y = 4 \sin t$, $t = \frac{3\pi}{4}$

Find the horizontal tangents of the curve.

23) $y = x^4 - 18x^2 - 4$

The figure shows the velocity v of a body moving along a coordinate line as a function of time t . Use the figure to answer the question.



25) Find $\frac{dy}{dx}$ for $y = (2x^3 + 1)^3(x^4 + 1)^5$

26) Find $\frac{dy}{dx}$ for $y = \frac{(2x^3 + 1)^3}{\sqrt[3]{x^2}}$