**Advanced Algebra With Financial Applications**
**COURSE PURPOSE**

Advanced Algebra With Financial Applications is a mathematical modeling course that is algebra-based, applications-oriented, and technology-dependent. The course addresses college preparatory mathematics topics from Advanced Algebra, Statistics, Probability, under six financial umbrellas: Banking, Investing, Credit, Employment and Income Taxes, Automobile Ownership, and Independent Living. The course allows students to experience the interrelatedness of mathematical topics, find patterns, make conjectures, and extrapolate from known situations to unknown situations. The mathematics topics contained in this course are introduced, developed, and applied in an as-needed format in the financial settings covered. Students are encouraged to use a variety of problem-solving skills and strategies in real-world contexts, and to question outcomes using mathematical analysis and data to support their findings. The course offers students multiple opportunities to use, construct, question, model, and interpret financial situations through symbolic algebraic representations, graphical representations, geometric representations, and verbal representations. It provides students a motivating, young-adult centered financial context for understanding and applying the mathematics they are guaranteed to use in the future, and is thusly aligned with the recommendations of the Common Core State Standards, as stated in this excerpt:

*"...all students should be strongly encouraged to take math in all years of high school. ...An array of challenging options will keep math relevant for students, and give them a new set of tools for their futures..."* From the Common Core State Standards

Advanced Algebra With Financial Applications offers 11th and 12th grade students an opportunity to view the world of finance through a mathematical lens. The topics were developed using the Common Core State Standards in Mathematics and the NCTM Curriculum and Evaluation Standards. The mathematical formulas, functions, and pictorial representations used assist students in making sense of the financial world around them and equip them with the ability to make sound financial decisions.

The overarching purpose of the course is to develop the type of mathematically proficient students addressed in this excerpt from the Common Core State Standards for Mathematics.

*Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose*.

Advanced Algebra With Financial Applications builds strength in reasoning and number sense, because the real-world applications demand that solutions make sense. Through contextual problem solving and the mathematical modeling of real situations, the course gives the students the motivation to persevere through routine and non-routine problems, and as a result, develop strength and confidence in their mathematics ability.

**Advanced Algebra With Financial Applications**
**COURSE OUTLINE**

## Chapter 3: Banking Services

In this unit, students use exponential functions to compute compound interest and compare it to simple interest. They derive formulas and use iteration to compute compound interest. They apply their findings to short-term, long-term, single deposit and periodic deposit accounts.

### Mathematics Topics

- Exponential functions
- Exponential growth and decay
- Solving exponential equations
- Using inductive reasoning

### Mathematics Learning Goals

- Students will use the simple interest formula $I = PRT$ and using inverse operations to solve for all four variables.
- Students will compute compound interest with and without the formula.
- Students will be able to identify $y = ax^b$ as exponential decay when x < 1.
- Students will be able to identify $y = ax^b$ as exponential growth when x > 1.
- Students will model a geometric series of the type $\sum_{b=0}^{n-1} ax^b$ .
- Students will graph exponential functions of the type $y = ax^b$

## Chapter 4: Consumer Credit

Becoming familiar with credit terminology and regulations is critical in making wise credit decisions. Credit comes at a price and in this unit students learn how to use mathematics to make wise credit choices that fit their needs, current financial situation, and future goals.

### Mathematics Topics

- Algebraic proportions
- Linear, quadratic, cubic, and exponential equations
- Exponential growth and decay
- Regression equations
- Inverse function of an exponential equation
- Logarithms
- Summation notation

### Mathematics Learning Goals

- Students will create, evaluate, interpret and solve algebraic proportions.
- Students will model situations using linear, quadratic, cubic, and exponential equations.
- Students will determine the curve of best fit using linear, quadratic, or cubic regression equations.
- Students will create, use, and interpret exponential growth and decay equations that model given situations.
- Students will create and use algebraic formulas and apply them for use in spreadsheets.
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**Chapter 5: Automobile Ownership**

Various functions, their graphs, and data analysis can be instrumental in the responsible purchase and operation of an automobile.

**Mathematics Topics**

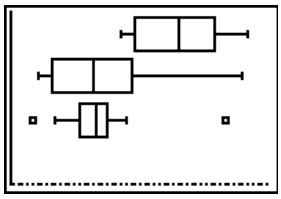
- Exponential/linear systems of equations
- Piecewise functions
- Graphs of piecewise functions
- Systems of linear equations
- Frequency distributions
- Stem-and leaf plots
- Modified box-and-whisker plots
- Measures of dispersion
- Quartiles
- Interquartile range
- Outliers of a frequency distribution

**Mathematics Learning Goals**

- Students will model exponential depreciation as $y = Px^b$ .where P is the purchase price and x < 1, and compare the depreciation to an increasing linear expense function.
- Students will transform raw data into a frequency distribution.
- Students will create and interpret stem and leaf plots and side-by-side steam plots such as

| | | |
|---|---|---|
| 9 8 1 1 1 1 | 87 | 1 2 2 |
| 3 2 | 88 | 2 4 6 7 |
| 7 5 4 | 89 | 1 3 |
| 7 6 6 6 6 5 | 90 | 2 7 7 7 |

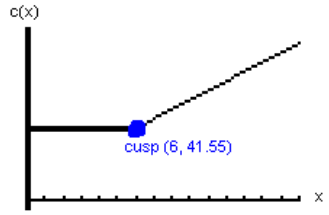
- Students will create and interpret side-by-side, modified box and whisker plots as shown:



- Students will compute measures of dispersion $R = x_H - x_L$ and $IQR = Q_3 - Q_1$.
- Students will compute $Q_1$, $Q_2$, $Q_3$, and $Q_4$ manually and with the graphing calculator.
- Students will compute boundaries for outliers using the expressions $Q_1 - 1.5(IQR)$ and $Q_3 + 1.5(IQR)$.
- Students will compute and interpret percentiles.
- Students will create and interpret piecewise (split) functions of the form

$$c(x) = \begin{cases} 38 \text{ when } x \le 4 \\ 38 + 6.25(x - 4) \text{ when } x > 4 \end{cases}$$

- Students will determine the domains of a piecewise function from verbal situations.
- Students will graph piecewise functions using mutually exclusive domains.
- Students will determine the cusp of a piecewise function at a change in slope such as




- Students will use multi-variable square root functions such as the skid length $S = \sqrt{30Dfn}$.
- Students will determine the reaction distance using the formula $RD = 0.75\left(\frac{5280s}{60^2}\right)$.
- Students will compute braking distance using the formula $BD = 5(.1s)^2$.
- Students will compute total stopping distance using the formula

$$TSD = 0.75\left(\frac{5280s}{60^2}\right) + 5(0.1s)^2.$$

- Students will compute distance, rate and time using $D = RT, R = \frac{D}{T}$, and $T = \frac{D}{R}$.
- Students will compute miles per gallon and distance using the formula $D = MPG(G)$.
- Students will use geometry theorems involving chords intersecting in a circle and radii perpendicular to chords to determine yaw mark arc length.
- Students will find the radius $r = \frac{C^2}{8M} + \frac{M}{2}$ where C is chord length and M is middle ordinate
- Students will compute arc lengths.
- Students will use dilations $D_k$ to transform formulas between the English Standard and Metric measurement systems.
- Students will adapt all algebraic formulas from the chapter for use in spreadsheets.

**Chapters 6-7: Employment and Income Taxes**

Many Internal Revenue Service and Social Security Administration regulations can be modeled by using linear and polygonal functions that have different slopes over different domains. Line-by-line instructions for IRS forms can also be algebraically symbolized.

**Mathematics Topics**

- Point-slope form of linear equations
- Jump discontinuities
- Continuous functions with cusps
- Slope
- Compound inequality notation
- Piecewise functions
- Interval notation
- Percent increase and decrease
- Data analysis
- Algebraic modeling

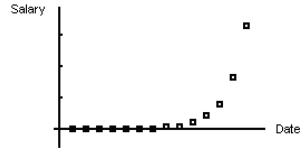
**Mathematics Learning Goals**

- Students will identify continuous and discontinuous functions by their graphs.
- Students will interpret jump discontinuities.

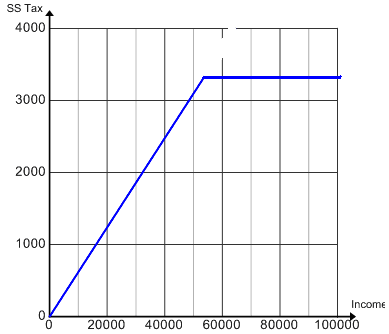
- Students will determine and interpret domains of piecewise functions of the forms

$$r(x) = \begin{cases} 29.95 \text{ if x is an integer and } x \le 2 \\ 29.95 + 14(x-2) \text{ if x is an integer and } x > 2 \end{cases} \qquad c(x) = \begin{cases} 0.20x \text{ when } 0 \le x < 750 \\ 0.22x \text{ when } 750 \le x \le 1{,}000 \\ 0.25x \text{ when } x > 1{,}000 \end{cases}$$

- Students will graph exponential pay schedules such as




- Students will graph piecewise functions with cusps such as




- Students will compute measures of central tendency and rational functions such as

$$a(x) = \frac{40r + 1.5tr}{t + r}.$$

- Students will express percent increases and decreases as rational functions.
- Introducing point-slope form $y - y_1 = m(x - x_1)$ and converting it to slope-intercept form $y = mx + b$.
- Students will translate verbal expressions into literal rational, exponential, and linear equations.
- Students will convert point-slope form to slope-intercept form of a linear equation.
- Students will write equations in point-slope form.
- Students will model algebraically a tax schedule of the form:

**Schedule Y-1—** If your filing status is **Married filing jointly** or **Qualifying widow(er)**

| If your taxable income is: | | The tax is: | |
|---|---|---|---|
| *Over—* | *But not over—* | | *of the amount over—* |
| $0 | $16,050 | ..... 10% | $0 |
| 16,050 | 65,100 | $1,605.00 + 15% | 16,050 |
| 65,100 | 131,450 | 8,962.50 + 25% | 65,100 |
| 131,450 | 200,300 | 25,550.00 + 28% | 131,450 |
| 200,300 | 357,700 | 44,828.00 + 33% | 200,300 |
| 357,700 | ..... | 96,770.00 + 35% | 357,700 |

- Students will create and interpret piecewise functions of the form

$$f(x) = \begin{cases} 0.10x & 0 < x \leq 16{,}050 \\ 1{,}605 + 0.15(x - 16{,}050) & 1{,}605 < x \leq 65{,}100 \\ 8{,}962.50 + 0.25(x - 65{,}100) & 65{,}100 < x \leq 131{,}450 \\ 25{,}550 + 0.28(x - 123{,}700) & 131{,}450 < x \leq 200{,}300 \\ 44{,}828 + 0.33(x - 200{,}300) & 200{,}300 < x \leq 357{,}700 \\ 96{,}770 + 0.35(x - 357{,}700) & x > 357{,}700 \end{cases}$$

where f(x) represents the tax liability function for taxpayers using a given tax schedule with taxable incomes on a given domain

- Students will graph piecewise functions of the form

$$f(x) = \begin{cases} y = 0.10x & 0 < x \leq 16{,}050 \\ y = 0.15x - 802.5 & 16{,}050 < x \leq 65{,}400 \\ y = 0.25x - 7{,}312.5 & 65{,}100 < x \leq 131{,}450 \end{cases}$$

- Students will determine the cusps of piecewise functions from the function notation.
- Students will interpret the graphs, slopes, and cusps of continuous polygonal functions with multiple slopes and cusps.
- Students will adapt all algebraic formulas in the unit for use in spreadsheets.

Advanced Algebra With Financial Applications

# KEY ASSIGNMENTS

The Key Assignments presented in this section are well-aligned with the CCSS Standards for Mathematical Practice. The assignments are all verbal problem solving activities that relate to the unit being studied. Students must represent the verbal situation symbolically, manipulate those symbols to arrive at an answer, and then interpret that answer in the context of the problem. This offers students opportunities to make sense of quantities and their relationships within those problem-solving settings through multiple representations. Students can approach, access, and deconstruct the necessary mathematics using handheld graphing utilities, manipulatives, spreadsheets, and/or software. The assignments throughout this course require students to attend to precision in their responses both in the computational and algebraic fluency required to arrive at those answers and in the units used to contextualize the answers.

The prevalence of mathematical modeling assignments allows students to practice seeking out mathematical structure in what may seem to them to be an unstructured situation. Identifying and exploiting the structure leads students to a richer understanding of the themes and regularities that are present in the real world. Students make tables, find patterns, and offer conjectures based on the patterns. This form of inductive reasoning is a cornerstone of mathematical thinking. The assignments and other course-related activities optimize students' exposure to extrapolating what they have learned to routine and non-routine mathematically-dependent situations they encounter in their futures.

Most assignments require the student to prepare a presentation on their finished work. This can be a PowerPoint show, a webinar, a poster presentation, or a presentation using transparencies. The student audience gets to critique the presentation, ask questions, and make comments, in a firmly established, constructive, positive "safe" zone. The presentation is graded, and the quality of student critiques and comments can also be graded.

## Creating the Tax Worksheet

**Mathematics:** Domains, piecewise functions, linear functions and graphs, point-slope form, slope-intercept form, graphs with cusps.
**Mathematics Learning Goals:** To derive the slope-intercept form used on the IRS tax worksheet by translating tax tables into piecewise functions.

The tax tables give taxpayers a function in which the independent variable is the taxable income and the dependent variable is the tax. It is convoluted and has confused taxpayers for years. Within the last decade, the IRS created a worksheet that uses the slope-intercept form of the equations of a line to simplify calculations for the taxpayer. In this Key Assignment, students interpret the IRS Schedule, express the domains using compound inequality notation, and create the piecewise function that models the IRS intentions. They then covert this function, which is a translated version of point-slope form, into the slope-intercept form to create the tax worksheet.

## Graphing the FICA Tax Function

**Mathematics:** Piecewise functions, slope, cusps, linear equations
**Mathematics Learning Goals:** To use graphs to compare the FICA tax longitudinally over a prescribed number of years.

Students look up the FICA tax percents, and maximum taxable incomes to create piecewise functions for each of the last six years. They compute the maximum FICA tax, and graph all six years on the same axes, and use the graph to write a paragraph on what has happened to FICA taxes over those years. They discuss the significance of the coordinates of the cusp. They do the same for the tax years 1981-86, and compare the last six years to the years 1981-1986. The assignment is replicated using the Medicare tax percent.

## Automobile Ownership

**Key Assignment 4.1: Using Statistics to Negotiate Auto Transactions**

**Mathematics:** Bivariate data, **c**orrelation, regression, mean, median, mode, quartiles, interquartile range, outliers, modified box-and-whisker plots, stem-and-leaf plots, frequency distributions, scatterplots.
**Mathematics Learning Goals:** To use measures of central tendency and measures of dispersion to mathematically negotiate the buying and/or selling of an automobile.

Students choose a make, model and year for an automobile. They use the Internet and newspaper classified ads to find 10-20 of those cars for sale. They get the price of the car and the mileage it has. They construct modified box-and-whisker plots and describe the frequency distribution. They pair each car's price with its mileage to create a scatterplot. They classify the association as positive or negative. They find the regression line and correlation coefficient and interpret the relationship as strong, moderate or weak, and discuss its linearity. Their results are presented to the class via PowerPoint presentation or poster presentation.

**Automobile Cost and Depreciation**

**Mathematics:** Exponential regression, **g**raphing linear and exponential functions, rational functions, linear/exponential systems, systems of linear equations, slope-intercept form.
**Mathematics Learning Goals:** To use graphing techniques to compare the value of a car to the expense of purchasing it throughout its lifetime.

Using the monthly payment rational function, students graph the cost C of purchasing a new car, using the down payment as the y-intercept, and the monthly payment as the slope. They then investigate three types of depreciation: straight-line, exponential, and historical bath tub graphs. They graph the cost and depreciation functions on the same set of axes to find the month at which the total cost C of owning the car surpasses its value V as it depreciates. They identify and interpret the domains on which $C > V$ and $C < V$.

**The Physics of Driving**

**Mathematics:** Quadratic equations, radical functions, arc length, geometry of the circle.
**Mathematics Learning Goals:** To use the mathematics listed to determine braking distances and to gather data from accidents scenes.

Students use formulas to determine reaction distance, braking distance, and figure out the speed a car was going based on its skid marks. The braking-distance formula is a quadratic function, with speed as the independent variable. The skid speed formula is an irrational function that has three independent variables. Students also use the geometry of the circle to compute the radius of a given yaw mark, which is a curved skid mark, and use the radius and friction factor to find the speed the car was going when it began to skid. The students then prepare a PowerPoint or poster presentation for the driver's education class in their school.

**Consumer Credit**

**Key Assignment 5.1: Can I Afford This Loan?**

**Mathematics:** Exponential functions, logarithmic functions, system of exponential and linear functions, modeling, graphical interpretation
**Mathematics Learning Goals:** To use three modalities to determine the affordability of a loan: exponential formula evaluation, logarithmic formula evaluation, and interpreting an exponential/linear system. To use technology (graphing utility and/or spreadsheet) to make the determinations required and justify their responses.

Students are given a scenario in which a family must make a decision about the affordability of a loan based on the principal, the loan-length, the APR and the maximum affordable monthly payment the family is able to make towards loan debt reduction. Students determine the affordability of the loan in three different ways: using the monthly payment

function, interpreting the graphs of the system of equations defined by the exponential monthly payment function and the linear maximum affordable monthly payment, and using the logarithmic loan length function. They are then asked to construct two spreadsheets: a monthly payment spreadsheet that charts the monthly payment as loan length time varies from 1 to 20 years, and a loan length spreadsheet that charts time as monthly payments vary from $100 to $1000. Finally, students must write up a summary analysis for this situation explaining how the algebraic modeling by the spreadsheet formulas supports their prior work.

**Mathematically Modeling a Credit Card Statement**

**Mathematics:** Algebraic modeling and spreadsheet formula creation
**Mathematics Learning Goals:** To algebraically model a month of activity on a person's credit card.

Students create a 21-day credit calendar that depicts algebraic representations of daily balances based upon an opening balance of Y dollars, an X-dollar purchased on the 8th day, a Z dollar payment on the 13th day, and a W-dollar purchased on the 20th day. Using these representations from the calendar, they write algebraic expressions for the sum of the daily balances, the average daily balance, and the finance charge for this 21-day period given that the APR on this credit card is P%. Students then create a spreadsheet that models the situation described above and test their spreadsheet for a given data set.

Advanced Algebra With Financial Applications

## INSTRUCTIONAL METHODS AND STRATEGIES

The instructional strategies used throughout this course are varied, targeted, and rooted in the CCSS Standards for Mathematical Practice. Just as the Standards are interrelated, the methods used in this course are. Together, the practices referenced in this section serve to build mathematical confidence, interest and strength.

The model problems and applications generally graduate in difficulty level, allowing the teacher to **differentiate instruction**. Since abstract reasoning can be difficult for many students, the instructions are graduated so students can grasp the higher level skills by meeting them step-by-step. This strategy allows student and teacher to identify the exact juncture at which the student is having difficulty. This makes diagnostics and intervention more pointed.

There is much opportunity for **experiential learning**. Projects require the students to get out in the field and meet with brokers, bankers, local businesses, etc. Guest speakers at several junctures bring the outside world right into the classroom. Students act as moderators and compile questions for the guest speaker. For some projects, data is gathered and statistically analyzed. Students present their work to the class, and they field questions and comments from their classmates.

**Technology** plays a key role in the development of topics. The graphing calculator is a daily tool, and its algebraic and graphing features are extensively used. Spreadsheets appear in every unit so students can model situations using algebra and technology.

Advanced Algebra With Financial Applications

# ASSESSMENT METHODS

A variety of formative and summative assessment methods are used throughout Advanced Algebra with Financial Applications in order to assess student learning. The assessments are aligned with the course purpose and the instructional strategies used, and with the Common Core Standards for the development of mathematically proficient students. In the activities listed below, students are offered assessment opportunities to address mathematics as a sense-making tool, problem solve, reason, construct arguments, offer mathematics-justified critiques of arguments, , model, use appropriate tools, attend to precision, look for and make use of structure, and look for and express regularity in repeated reasoning. The assessment grading percentages contributing to the student's quarter course grade are offered in parentheses next to the assessment name.

## FORMATIVE ASSESSMENTS (30%)

### CLASS PARTICIPATION (15%)

- **Do Now Activities** are assessments that can be used as a vehicle for the teacher to determine whether students have acquired skills, strategies, and content necessary for subsequent work in a topic. This diagnostic feature allows the teacher to adjust the lesson accordingly, if entry conditions are not fully met.

- **Check Your Understanding** problems are offered to students immediately after the teacher has introduced a new concept or procedure. These problems offer students and teacher alike an immediate assessment opportunity that is confined to the single new skill just addressed. The teacher can adjust the lesson to follow based upon review of these problems.

- **Extend Your Understanding** problems are more advanced problems that use the concepts and procedures just learned and take them to another level. These can be offered to all students or differentiated for selected students depending on the nature of the problems.

- **Ticket to Leave Activities** are ungraded activities that offer the teacher an opportunity to determine the level of understanding students acquired on the skills, strategies, and content of the day's lesson. These activities can be used by the teacher to adjust the following day's lesson.

- **Direct and Indirect Teacher Questions** are immediate formative methods of assessing students' understanding. In-class discussion is a critical part of Advanced Algebra with Financial Applications. The teacher should initiate discussion through focused questioning.

- Through the **Exploration of Essential Questions** (one per lesson), the teacher assesses student understanding both pre-instruction and post-instruction. The essential question is offered to the students at the beginning of the first lesson on the topic and a discussion ensures. That same essential question is revisited during the instruction and/or post-instruction to assess student growth and learning.

- Reading and writing are an essential part of Advanced Algebra with Financial Applications. Teachers will use **written and oral response to reading** (from the textbook, newspapers, magazines, Internet, brochures, laws, etc.) as a way of assessing understanding. Some writing activities will offer students an opportunity to interpret data that is displayed in a pictorial representation. Based upon the data, they are asked to write a short, newspaper-type story centered on the graph. There is one such activity for each chapter.

### HOMEWORK (15%)

- **Homework Assignments** are a daily evaluation and reflection device for both student and teacher. The level of proficiency with the homework questions should allow the teacher the opportunity to adjust the lesson as needed. The homework acts as a barometer for students, so they can formulate questions, and attempt problems on their own.

**SUMMATIVE ASSESSMENTS (70%)**

- **Lesson-Opener Quizzes** are short, graded, cumulative assessments that can test for prerequisite skills and/or mastery of recently taught material. These assessments are averaged and count as one full-period exam grade.

- **Full-Period Exams** are graded summative assessments that test student acquisition of skills, strategies, and content.

- **Experiential Learning** activities are project-based assessment tools that are offered to students as long-term assignments. Students are asked to do research in a variety of forms and formats in order to accomplish a task that is related to the skills, strategies, and content covered in the chapter. Their projects can be submitted in print, electronic, or presentation format. Precision and accuracy will be scrutinized during their presentations as well as the ability to use mathematical tools appropriately and strategically. Each project is valued as a single full-period exam grade.