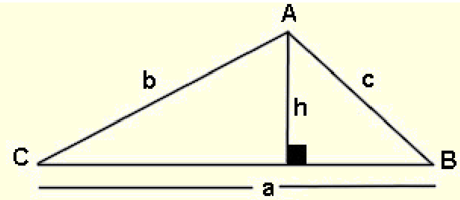
Area and Law of Sines

We are all familiar with the formula for the area of a triangle, $A = \frac{1}{2}bh$, where b stands for the base and h stands for the height drawn to that base.



(the lettering used is of no importance)

Express the area of this triangle using a as the base.

Express the ratio $\sin C$ in terms of h.

Use substitution to write a formula for the area of <u>any</u> triangle using trigonometry!

1. In ΔABC, $m\angle A = 120°$, $b = 10$, and $c = 18$. What is the area of ΔABC to the *nearest square inch*?

2. The sides of a parallelogram measure 10 cm and 18 cm. One angle of the parallelogram measures 46 degrees. What is the area of the parallelogram, to the *nearest square centimeter*?

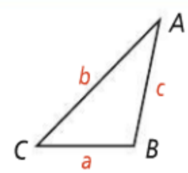
3. The two sides and included angle of a parallelogram are 18, 22, and 60°. Find its exact area in simplest form.

take note

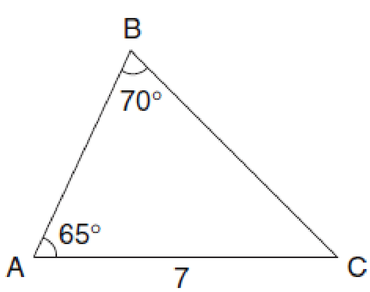
Theorem **Law of Sines**

In any triangle, the ratio of the sine of each angle to its opposite side is constant. In particular, for $\triangle ABC$, labeled as shown,

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}.$$



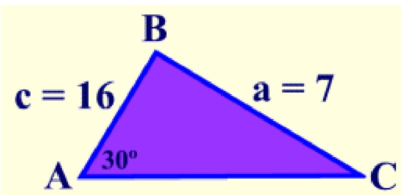
4. In the accompanying diagram of ΔABC, $m\angle A = 65°$, $m\angle B = 70°$, and the side opposite vertex *B* is 7. Find the length of the side opposite vertex *A.*



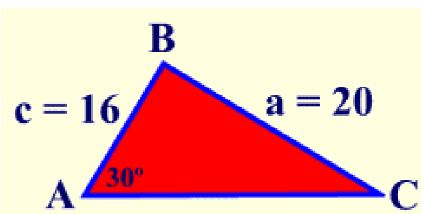
The Ambiguous Case

By definition, the word **ambiguous** means *open to two or more interpretations.* Such is the case for certain solutions when working with the Law of Sines. If you are given two angles and one side (ASA or AAS), the Law of Sines will nicely provide you with ONE solution for a missing side. Unfortunately, the Law of Sines has a problem dealing with SSA. If you are given two sides and one angle (where you must *find an angle*), the Law of Sines could possibly provide you with <u>0, 1, or 2 solutions</u>.

5. In ΔABC, $a = 7$, $c = 16$, and $m\angle A = 30°$. How many distinct triangles can be drawn given these measurements? Find all possible measures for $\angle B \text{ and } \angle C$.



6. In ΔABC, $a = 20$, $c = 16$, and $m\angle A = 30°$. How many distinct triangles can be drawn given these measurements? Find all possible measures for $\angle B \text{ and } \angle C$.



7. In ΔABC, $a = 10$, $b = 16$, and $m\angle A = 30°$. How many distinct triangles can be drawn given these measurements? Find all possible measures for $\angle B \text{ and } \angle C$.